LASER STRIPPING H\(^{+}\) CHARGE EXCHANGE INJECTION BY FEMTOSECOND LASERS

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Abstract

A new method of H\(^{+}\) laser assistant charge exchange injection using femtosecond laser pulses is considered. The existing method uses a divergent laser beam that allows compensation for the angular and momentum spread of the stripped beam. The femtosecond laser pulse has a similar property that can cover the spread and yield efficient laser stripping. Results of simulations with realistic femtosecond laser and H\(^{+}\) beam parameters are discussed. The proposed method may have some benefits for particular technical conditions over other methods.

INTRODUCTION

Femtosecond lasers may be applied for H\(^{+}\) charge exchange injection method or laser stripping. Laser stripping has been actively developed at the SNS project over the past decade. Theoretical investigation began in [1]. Practical application has been proven for stripping of short (6 ps) pulses [2] and for long (10 us) pulses of H\(^{+}\) [3]. The second step of the three-step laser stripping scheme [1] considers resonant excitation of the H\(^{0}\) beam by laser at the incidence angle \(\alpha\):

\[
\omega_0 = \gamma_p(1 + \beta_p \cos(\alpha))\omega
\]

where \(\beta_p\) and \(\gamma_p\) are relativistic factors of the H\(^{0}\) beam with longitudinal momentum \(p\). Equation (1) represents a precise resonant condition for monochromatic laser frequency \(\omega\) and discrete atomic excitation frequency \(\omega_0\). Longitudinal momentum spread \(\delta p\) of a realistic beam spreads out particles from resonance particle with design momentum \(p\). As a result, most particles are not excited and stripped. In this way, longitudinal momentum spread complicates laser stripping and presents a challenging practical problem. This section presents a qualitative review of the methods of compensating \(\delta p\) in terms of (1), skipping the complicated mathematical mechanism of laser-bunch interaction.

Paper [1] proposed the use of a divergent laser beam with an angle of \(\delta \alpha\) to cover the spread \(\delta \gamma(\delta p)\), \(\delta \beta(\delta p)\). A laser beam with an angle of about 0.5 mrad provides good resonance conditions for all the particles of the bunch. This method has been tested experimentally [2, 3].

Another method of \(\delta p\) compensation has been proposed in [4] and developed in [5]. It involves broadening the discrete atomic resonance frequency \(\omega_0\) by applying an electric field in the particle’s rest frame or a magnetic field in the laboratory frame. All off-resonance particles can be excited by using wide continuum atomic resonance \(\delta \omega_0\) when a high energy atom is emerged into a strong magnetic field in the laboratory frame.

This paper discusses another method of momentum beam compensation which involves broadening the laser frequency \(\omega\) in terms of (1). The picosecond laser pulse that has been considered in [2] or [3] has a narrow band frequency \(\omega\) which may be broadened by using femtosecond laser pulses. The spectrum \(f(\omega')\) of finite laser pulse

\[
E(t) = E_0 e^{-\frac{t^2}{4\sigma^2}} \cos(\omega t) = \int_{-\infty}^{\infty} f(\omega')\cos(\omega' t)d\omega'
\]

with pulse length \(\sigma\) (in terms of energy) can be calculated by Fourier transform

\[
f(\omega') = \frac{E_0 \sigma e^{-\sigma^2(\omega'+\omega)^2}}{2\sqrt{\pi}} \left(1 + e^{\frac{4\sigma^2\omega_0\sigma^2}{\omega_0}}\right)
\]

The RMS width \(\sigma_\omega\) of the function is estimated to be

\[
\sigma_\omega = \frac{1}{\sigma} \sqrt{\frac{1}{2} \frac{1}{\pi} \frac{0.42}{\sigma}}
\]

A shorter pulse has a wider spectrum. The required laser pulse width \(\sigma\) may be estimated by differentiating \(\omega\) over momentum \(p\) and equating \(d\omega_0\) to \(\sigma_\omega\). The SNS accelerator has a relative longitudinal momentum spread \(\Delta p/p \approx 10^{-4}\) and the laser pulse width \(\sigma\) is estimated to be about 1 ps. This rough estimate indicates that a sub-picosecond or femtosecond laser pulse is required to compensate the longitudinal momentum spread for high efficiency excitation.

All three methods have their relative advantages and disadvantages and each method can become more convenient for accelerator with particular parameters. The divergent laser beam method allows for adjustment of the incidence angle \(\alpha\) for beam energy in a wide range; however, it requires precise laser optics tuning. The atomic broad shape resonance method does not require tuning the laser beam optics but may require a more complicated (or simplified) magnet system for stripping the H\(^{+}\) beam. It also requires more beam energy [5]. The third method differs from the others by using femtosecond lasers. The H\(^{+}\) bunch usually has a multi-picosecond width that requires about the same longitudinal width as the laser to achieve adequate interaction overlap. For this reason, a femtosecond laser pulse is difficult to use for stripping the H\(^{+}\) bunch at a nonzero incidence angle of \(\alpha\). Adequate overlap with femtosecond pulse is only achieved by using head-on interaction or setting the angle at 0. As existing
powerful femtosecond lasers have only one or two wavelengths, laser stripping is only possible for fixed $H$ energy defined by (1) where $\alpha = 0$. Another disadvantage to this method is the more complicated scheme of stripping with head-on interaction.

**MODEL OF INTERACTION**

This section presents a more detailed physical model for the interaction of a femtosecond laser pulse with a $H^0$ beam. A $H^0$ atom interacting with the femtosecond laser pulse can be characterised by the time-dependant, quantum-mechanical wave function $\Psi(t) = C_1(t)\psi_1 + C_2(t)\psi_2$. The atom is excited from the ground state $\psi_1$ to an excited state $\psi_2$. The coefficients $|C_1|^2$ and $|C_2|^2$ represent the probability of the atom to be in the corresponding states.

![Diagram](image)

Figure 1: Interaction of atomic hydrogen with laser beam in general case for $\alpha \neq 0$ in (1).

As found in all previous studies, the wave-function and the excitation (or stripping) efficiency $|C_2|^2$ can be found by solving the time-dependant Schrodinger equation:

$$i\hbar \Psi = (\hat{H}_0 + V_{\text{laser}})\Psi$$  \hspace{1cm} (5)

This equation represents the dynamic of an electron in a proton field $H_0$ with an external laser field $V_{\text{laser}}$.

For the sake of simplicity, using atomic units and eliminating derivations, the main equations for laser-$H^0$ interaction in the particle’s rest frame are expressed thusly:

$$\dot{C}_1 = i\mu E_0 \frac{e^{-i\omega t}}{2} C_2$$
$$\dot{C}_2 = i\mu E_0 \frac{e^{i\omega t}}{2} C_1$$  \hspace{1cm} (6)

The electric component of the femtosecond laser pulse is written in a complex form with the Gaussian profile:

$$E(t) = E_0 e^{-\frac{t^2}{4\sigma^2} + i\omega t}$$  \hspace{1cm} (7)

The parameters of the laser electric field component $E_0$, its frequency $\omega$, and the pulse width $\sigma$ must be transformed from the laboratory frame to the particle’s rest frame using relativistic transformations. This section discusses equation (6) in detail and presents a quantitative explanation of how the short laser pulse would achieve high efficiency excitation of the whole bunch.

The $H^0$ particle remains in the ground (non-excited) state before interaction $C_1(\infty) = 1$, $C_2(\infty) = 0$ and transitions to the atomic state $C_2(\infty)$ after interaction with the laser. Consequently, it is convenient to reassign the time variable, as it is not needed for $C_2(x)$:

$$\dot{C}_1 = i\mu E_0 \frac{e^{-i\Delta \omega t}}{2} C_2$$
$$\dot{C}_2 = i\mu E_0 \frac{e^{i\Delta \omega t}}{2} C_1$$  \hspace{1cm} (8)

where $\Delta \omega = \omega_0 - \omega$ is an off-resonant term. The equation (8) may be solved analytically for the particle with the exact resonance condition $\omega_0 = \omega$ and the excitation probability may be written as:

$$|C_2(\infty)|^2 = \sin(\sqrt{\pi} \mu \sigma E_0)^2$$  \hspace{1cm} (9)

The particle may be excited to 100% only by particular laser peak power: $\sqrt{\pi} \mu \sigma E_0 = \pi/2$. The excitation condition (9) used in this study is a nonlinear, relatively high laser power and cannot be obtained by a cross section method. The laser peak power with amplitude of its electric component $E_0 = \sqrt{\pi}/(2\mu \sigma)$ is the nominal power for excitation of a design particle and will be used throughout this paper. Equation (8) is difficult to solve analytically for $\Delta \omega \neq 0$; however, it may be solved approximately for small $\Delta \omega$, similar to the perturbation method in quantum mechanics. Equation (8) may be written as

$$\begin{pmatrix} \dot{C}_1 \\ \dot{C}_2 \end{pmatrix} = A(t, \Delta \omega) \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$$  \hspace{1cm} (10)

and may be solved exactly for $\Delta \omega = 0$, which may be considered as zero approximation $C_1(0), C_2(0)$. First-order approximation may be found from the equation:

$$\begin{pmatrix} \dot{C}_1 \\ \dot{C}_2 \end{pmatrix} = A(t, \Delta \omega) \begin{pmatrix} C_1(0) \\ C_2(0) \end{pmatrix}$$  \hspace{1cm} (11)

where the excitation probability $|C_2|^2$ may be expressed in the following way:

$$|C_2|^2 = \left| \int_{-\infty}^{\infty} e^{i\omega t} + i\sigma \Delta \omega t - i^2 C_1(0) \, dt \right|^2$$  \hspace{1cm} (12)

Solution of (12) is beyond the scope of this paper.

Equation (8) visually demonstrates (without solving) that a short laser pulse may compensate off-resonant particles. An increased longitudinal momentum spread $\delta p$ results in an increased resonant spread $\delta \omega_0$, which can be compensated by a smaller laser pulse width $\sigma$ to make the excitation probability close to 1. The power of the laser pulse $Q$, which is proportional to $P_0 \sigma$ where $P_0$ is a peak
power, may be represented as a function of laser pulse width \( \sigma \) and \( \delta \omega_0 \), taking into account condition (9):

\[
Q \sim \sigma p_0 \sim \sigma E_0^2 \sim \frac{1}{\sigma} \sim \delta \omega_0 \sim \delta p_z
\]  

(13)

This relation shows that increased momentum spread requires increased laser power.

**THREE PARTICLES CALCULATION**

This section presents an example of the interaction of a femtosecond laser pulse with three \( H^0 \) particles of different parameters of longitudinal momentum: \( p, p+\Delta p, \) and \( p+2\Delta p \). Parameter \( \Delta p=10^{-4}p \) corresponds to a realistic parameter of longitudinal momentum spread taken from the SNS accelerator. The laser pulse is considered as a round Gaussian beam with a transverse RMS size of 1 mm and wavelength of 1064 nm. The energy of the design particle is calculated to be 3.22 GeV for excitation to the second excited state. Figures 2, 3, and 4 represent the dynamics of excited states \( |C_2|^2 \) in the particle’s rest frame for various laser pulse widths.

**Figure 2:** Excitation of three particles by the 300fs and 10 \( \mu \)J laser pulse in the particle’s rest frame. All particles with momentum \( p, p+\Delta p, \) and \( p+2\Delta p \) are excited.

**Figure 3:** Excitation of three particles by the 3000fs (or 3ps) and 1.0 \( \mu \)J laser pulse in the particle’s rest frame. Off-resonance particles with momentum \( p+\Delta p, \) and \( p+2\Delta p \) are not excited.

**Figure 4:** Excitation of three particles by the 30000fs (or 30ps) and 0.1 \( \mu \)J laser pulse in the particle’s rest frame. Off-resonance particles with momentum \( p+\Delta p, \) and \( p+2\Delta p \) are not excited.

The laser pulse energy is calculated for 100% excitation of the design particle with momentum \( p \). It should be noted that the laser pulse width in the particle’s rest frame shown in the figures is smaller than nominal width in the laboratory frame by factor of \( \gamma(1+\beta) \). The figures demonstrate that a femtosecond laser pulse of about 300fs can excite all particles within the given longitudinal momentum spread. Realistic beam excitation efficiency may be calculated by integration over beam distribution \( f(p,r) \):

\[
\int_{-\infty}^{\infty} |C_2(p_z, r)|^2 f(p_z, r) 2\pi r dr dp_z
\]

(14)

however, the result will be close to the three particle estimate.

Table 1 shows possible variants of laser wavelength, laser power, \( H^0 \) excitation state, and \( H^0 \) beam energy. The laser pulse width is 300fs.

**Table 1. Parameters of \( H^0 \) and Laser Pulse**

<table>
<thead>
<tr>
<th>Laser wavelength</th>
<th>Laser pulse energy</th>
<th>( H^0 ) beam energy</th>
<th>( H^0 ) excitation state</th>
</tr>
</thead>
<tbody>
<tr>
<td>1064 nm</td>
<td>10 ( \mu )J</td>
<td>3.22 GeV</td>
<td>2</td>
</tr>
<tr>
<td>1064 nm</td>
<td>61 ( \mu )J</td>
<td>3.97 GeV</td>
<td>3</td>
</tr>
<tr>
<td>1064 nm</td>
<td>175 ( \mu )J</td>
<td>4.24 GeV</td>
<td>4</td>
</tr>
<tr>
<td>800 nm</td>
<td>10 ( \mu )J</td>
<td>2.22 GeV</td>
<td>2</td>
</tr>
<tr>
<td>800 nm</td>
<td>61 ( \mu )J</td>
<td>2.78 GeV</td>
<td>3</td>
</tr>
<tr>
<td>800 nm</td>
<td>175 ( \mu )J</td>
<td>2.98 GeV</td>
<td>4</td>
</tr>
</tbody>
</table>

This table shows an estimate of the minimum laser pulse energy 10 \( \mu \)J required for stripping the second excited state. The second excited state with a beam energy of 3.22 GeV may be easily stripped by a regular (non-superconducting) magnet. The estimated femtosecond

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laser pulse width and energy may be achieved with existing femtosecond laser technology [6].

The experimental scheme of this method appears complicated due to the head-on interaction of the femtosecond laser and the H- beam. This is not discussed in this paper and requires further investigation.

**SUMMARY**

- A femtosecond laser pulse may be used for high efficiency stripping of a realistic beam with longitudinal momentum spread.
- An increased longitudinal beam spread would require more average laser beam power.
- Femtosecond laser stripping may be applied only to a few discrete H- beam energies.
- Estimated required laser parameters appear achievable with current femtosecond laser technology.
- Femtosecond laser stripping scheme for real accelerator is not developed and requires additional investigation.

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**REFERENCES**


