MITIGATION OF NUMERICAL NOISE FOR BEAM LOSS SIMULATIONS

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Abstract

Numerical noise emerges in self-consistent simulations of charged particles, and its mitigation is investigated since the first numerical studies in plasma physics [1–3]. In accelerator physics, recent studies find an artificial diffusion of the particle beam due to numerical noise in particle-in-cell tracking [4], which is of particular importance for high intensity machines with a long storage time, as the SIS100 at FAIR [5] or in context of the LIU upgrade [6] at CERN. In beam loss simulations for these projects artificial effects must be distinguished from physical beam loss. Therefore, it is important to relate artificial diffusion to artificial beam loss, and to choose simulation parameters such that physical beam loss is well resolved. As a practical tool, we therefore suggest a scaling law to find optimal simulation parameters for a given maximum percentage of acceptable artificial beam loss.

HEAVY-ION BEAM LOSSES

The uncontrolled loss of charged particles is an important issue in high energy particle accelerators. For 1 GeV proton beams, it was found that 1 W/m is the maximum tolerable beam loss to allow hands-on-maintenance [7]. If more the energy is deposited, a worker would be exposed a too high dose during maintenance, and hence a health hazard.

However, this limit of energy deposition is only valid for 1 GeV proton operation. According estimates for heavy-ion machines were only found recently [8] by dedicated simulation studies in which a uniform beam loss along a beam pipe is considered. The pipe is irradiated for 100 days, while the effective dose rate was calculated four days after the radiation stopped. Then, the residual activity is compared to the residual activity caused by 1 GeV protons, in order to infer a scaling law for heavy ions. Using this scaling law, we estimate the maximum acceptable beam loss per run in the SIS100 for U^{+28} particle beams at different energies, see the Table 1.

Table 1: Maximum Acceptable Beam Loss

part. energy	energy dep.	# particles
200 MeV/u	75 W/m	$1.1 \cdot 10^{13}$
500 MeV/u	23 W/m	$1.3 \cdot 10^{12}$
1000 MeV/u	12 W/m	$3.4 \cdot 10^{11}$

The design goal of the SIS100 is a maximum of $5 \cdot 10^{11}$ particles of U^{+28} stored in the machine, such that only the loss of a full 1GeV/u Uranium beam depicts a hazard. For particle beams with less energy a complete, but uniform, beam loss can be tolerated.

Further, beam loss may cause a heating of superconducting structures, such that the material changes to the normal conducting phase. This may lead to a serious machine damage, or at least will require maintenance, and thus an interruption of beam time. It was found at the Large Hardron Collider (LHC) at CERN, that a nominal beam loss of the order of 10^{-6} corresponding to 10^6 protons can cause a magnet quench [9]. The limit on the nominal beam loss for the LHC is exceptionally small, because the beam energy and the and the intensity are very high compared to other machines. Simulation studies on the superconducting magnets of the SIS100 synchrotron at FAIR show that there is no risk of a magnet quench [10].

The lifetime of organic insulators and protection diodes in superconducting magnets is expected to give the most restrictive limit to beam loss for the SIS300 synchrotron. It was found in simulation studies that a maximum of 2 percent nominal beam loss can be tolerated for this machine [11].

In summary, the maximum acceptable beam loss varies greatly for various scenarios, such that different upper bounds are required for artificial beam loss in numerical simulations.

ARTIFICIAL BEAM LOSS

In the following chapter, we present an analytic model to predict artificial beam loss induced by numerical noise in particle-in-cell tracking. Beam loss occurs whenever the emittances of a single particle i are larger than the acceptance of the machine. A collimator allows controlled particle loss, as particles with large amplitudes can be removed without activating the accelerator structures. By adjusting the geometry of a collimator, the acceptance of a machine can be set to the required size. The rms emittance of a particle beam is given by

$$\epsilon_x = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle x x' \rangle^2}, \qquad (1)$$

and accordingly for the y-plane. Here, $\langle \cdot \rangle$ is the moment of the according coordinate, and

$$\langle x^2 \rangle = \sigma_x^2 \qquad \langle x'^2 \rangle = \sigma_{x'}^2 \qquad (2)$$

are the variances of the phase space coordinates, which quantify the beam size.

The rms emittance of a beam growths linearly in the presence of numerical noise, as long as numerical noise is weak and not correlated. The average emittance growth per integration step of length Δs was derived from a single particle

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model as [4]:

$$\frac{\Delta\epsilon_x}{\Delta s} \simeq \Lambda \frac{\sigma_x^2}{2\epsilon_x}\left(\frac{q\delta E_x}{m_o c^2\beta^2\gamma^3}\right)^2 \Delta s. \tag{3}$$

Here, Λ accounts for the particle density of the distribution, with $\Lambda = 1$ for K-V beams and $\Lambda = 0.5$ for Gaussian beams. Further, m_0 is the mass of the particle, c is the speed of light, and β, γ are the relativistic factors. The factor δE_x accounts for the precision of the Poisson solver, and can be found by a simple numerical investigation. We randomly initialize the particle distribution, and calculate the electric field in the center of the beam. We repeat this procedure n times, and we identify the standard deviation of the n samples with δE_x. This analysis has a low computational load compared to a full tracking simulation, and can be used to efficiently find optimal simulation parameters for long-term tracking simulation, as shown in Ref. [4, 12].

As the particle beam diffuses due to numerical noise, the single particles may exceed the acceptance in long-term tracking simulations. We therefore expect to observe numerical noise induced particle loss. In the following, we derive estimates for such artificial beam loss, and find a scaling law on simulation parameters. These scalings can be used to find optimum simulation parameters in terms of computational load for a maximum tolarable amount of artificial beam loss.

The general form of a particle distribution function for a beam after injection is given by

$$f(\epsilon_{x,i},\epsilon_{y_i})(s) = f\left(\frac{\epsilon_{x_i}}{\epsilon_x(s)} + \frac{\epsilon_{y_i}}{\epsilon_y(s)}\right), \tag{4}$$

where ϵ_{x_i} and ϵ_{y_i} are the single particle emittances. For an investigation on artificial beam loss, we have to consider that $f(\epsilon_x,\epsilon_y)(s)$ changes while tracking due to numerical noise. In the following we discuss artificial beam loss for the Gaussian distribution, and the Kapchinsky-Vladimirsky (K-V) distribution.

Gaussian Distribution

The Gaussian particle distribution is given by

$$f(\epsilon_{x,i},\epsilon_{y_i})(s) = \frac{1}{4\epsilon_x\epsilon_y} e^{-\frac{1}{2}\left(\frac{\epsilon_{x,i}}{\epsilon_x(s)} + \frac{\epsilon_{y_i}}{\epsilon_y(s)}\right)}, \tag{5}$$

such that the single particle coordiantes follow a Gaussian distribution function. The mathematical condition for single particles to hit a collimator is given by the inequality

$$\frac{x^2}{(3\sigma_x)^2} + \frac{y^2}{(3\sigma_y)^2} > 1, \tag{6}$$

or, in terms of emittances,

$$\frac{\epsilon_{x_i}}{9\epsilon_x} + \frac{\epsilon_{y_i}}{9\epsilon_y} > 1. \tag{7}$$

For illustration, we plot a sample Gaussian distribution in Fig. 1 together with the condition given by Eq. 7. Most

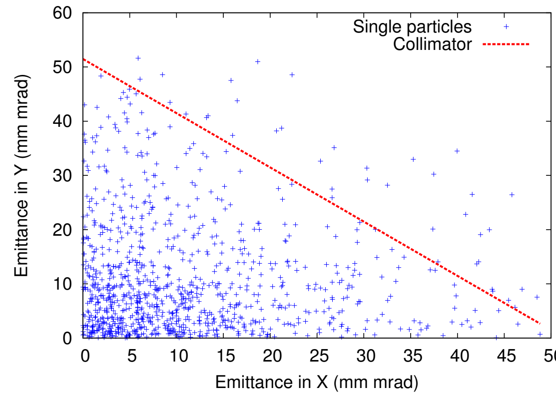

Figure 1: Distribution of single particle emittances (blue points) of a Gaussian beam, and the condition for collimation according to Eq. 7 (red line).

of the particles are initially not affected by the collimator. However, a grow of single particle emittance will slowly induce losses.

To estimate the amount of artificial particle loss, we develop a theoretical model where we consider beams with equal rms emittances, i.e. $\epsilon_x(s) = \epsilon_y(s) = \epsilon(s)$. We then find the condition for collimation as

$$\epsilon_{x_i}(s) + \epsilon_{y:i}(s) > 9\epsilon(s_0) \equiv A, \tag{8}$$

where A is the acceptance of the machine, set by the collimators. With this, we find the nominal beam loss as

$$\frac{\Delta N}{N} = 1 - \int_0^A d\epsilon_{x_i} \int_0^{A-\epsilon_x} d\epsilon_{y_i} f(\epsilon_{x_i},\epsilon_{y_i})(s). \tag{9}$$

We use Eq. 5, to derive

$$\frac{\Delta N}{N} = \left(\frac{A}{2\epsilon_x(s)} + 1\right) e^{-\frac{A}{2\epsilon_x(s)}}, \tag{10}$$

while the evolution of the rms emittance $\epsilon_x(s)$ is described by the scaling law Eq. 3. This model can be compared to previous models of beam loss [13, 14], where nominal beam loss was estimated by solving a diffusion equation via Bessel basis functions. For sufficiently large nominal emittance growth, it was found that

$$\frac{\Delta N}{N} = 1 - \exp\left(-\frac{\lambda_1}{4A}\left(\frac{\Delta\epsilon_x}{\Delta s}\right)s\right), \tag{11}$$

where λ_1 is the first zero of the Bessel function. We compare Eq. 10 and Eq. 11 in Fig 2, and find a significantly larger nominal beam loss for the model derived in this proceeding,. This discrepancy can be explained by the assumption of large emittance growth in Ref. [13], which refers to a beam shape given by the Bessel function J_0, while in our model a Gaussian distribution is assumed. Further, the self-consistent effect of the particle loss on numerical noise is not considered in our model. Therefore, our model is only valid for particle losses, where the distribution remains Gaussian, and the particle loss itself is not changing the strenght of numerical noise.

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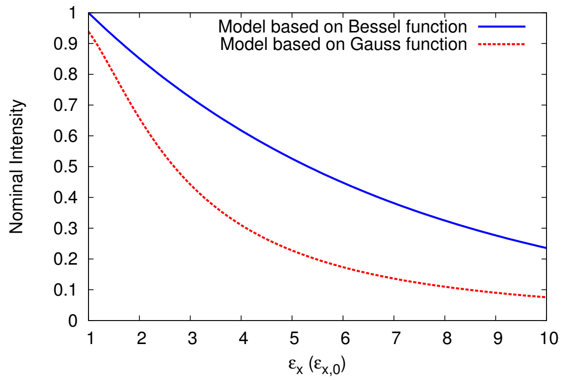

Figure 2: Comparision of models for beam loss caused by diffusion, where a Gaussian beam distribution is assumed (red line), or the distribution is approximated by the Bessel function J_0 (blue line).

K-V Distribution

The previously derived model for artificial beam loss is valid only for Gaussian beams. In the following, we investigate the artificial beam loss for particles beams described by the Kapchinsky-Vladimirsky (K-V) distribution [15], that is given by

$$f(\epsilon_{x_i}, \epsilon_{y_i})(s) \propto \delta\left(\frac{\epsilon_{x,i}}{2\epsilon(s)} + \frac{\epsilon_{y_i}}{2\epsilon(s)} - 1\right). \tag{12}$$

such that solely a single line $\epsilon_{x,i} = 2\epsilon - \epsilon_{y,i}$ is populated, as shown in Fig. 3.

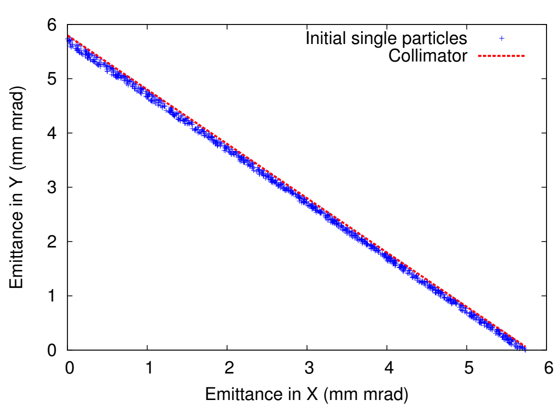

Figure 3: Distribution of single particle emittances (blue points) of a K-V beam, and the according condition for collimation (red line).

When placing the collimators at the edges of the beam, we find that even a weak numerical noise, and thus small artificial beam diffusion, may cause dramatic artificial beam loss in K-V beams

ESTIMATION OF OPTIMAL SIMULATION PARAMETERS

Previous results for Gaussian beams can be used to find optimal simulation parameters for beam loss studies. In the following, we discuss the case of beam loss simulations for the SIS100 at FAIR, but for simplicity assume a circular beam. The beam and simulation parameters are given as:

- beam size: $\sigma_x \simeq \sigma_y \simeq 7$ mm,
- tunes: $Q_x = 18.87$, and $Q_y = 18.74$,
- emittance: $\epsilon_x \simeq \epsilon_y \simeq 5.7$ mm mrad
- integration lenght: $\Delta s \simeq 5.703$ m
- ring length: $L = 1083.5$ m
- number of turn: $N_t = 2.0 \cdot 10^5$
- number of kicks: $N_s = 2 \cdot 10^5 \cdot 190$
- space charge tune shift: $Q_x \simeq Q_y \simeq -0.21$
- relativistic factors: $\beta = 0.56768$, $\gamma = 1.2147$
- particle: Uranium-238 in charge state 28

In the following, we consider an exemplary scenario, where the physics case requiers a maximum of 1 percent of artificial beam loss. We use Eq. 3 and Eq. 10, to find the condition for the minimum precision of our solver as

$$\langle \delta E_x \rangle \leq 28.9\ \mathrm{Vm}^{-1}. \tag{13}$$

In the following, we find optimal simulation parameters for the MICROMAP tracking library [16], that utilizes a two-dimensional spectral solver [17–19]. For the solver, we use as a first attempt $N_M = 1000$ macro-particles, and a mesh of $N_G \times N_G = 64 \times 64$ grid points. By analyzing $n = 1000$ samples of the electric field initializations, we find

$$\delta E_x \simeq 210\ \mathrm{Vm}^{-1}. \tag{14}$$

We thus have to increase the precision of the solver, by a factor of about 7 to limit the nominal beam loss to 1 percent, which requires $N_M \simeq 53.000$ macro-particles. If instead we consider a maximum of 0.1 percent of beam loss, our theory predicts an optimum number of macro-particles of $N_M \simeq 514.000$.

BEAM LOSS SIMULATIONS

In the following, we present beam loss simulations of coasting beams whose dynamics is distorted by strong numerical noise, and compare it to our analytical derivations. The tracking is performed with MICROMAP [16], while we use the beam and simulation parameters as listed in the previous chapter. The collimators are placed at $\pm 3\sigma_x$ and $\pm 3\sigma_y$, such that particles with larger amplitude are scraped within the first turns. As this effect is superposed with the artificial beam loss pattern, we normalize the beam intensity after 2000 turns. In order to resolve well the beam loss pattern, we repeated the simulation 15 times, and show the average loss pattern in Fig. 4.

We reach agreement on the model with the loss pattern for different number of macro-particles. In the following, we investating the beam loss pattern of a rms equivalent [20,21] K-V beam, for which the collimators are placed at positions slightly larger than the beam size. In Figure 5, we show the beam loss pattern for different number of macro-particles, and with the same beam and simulation parameters as for Gaussian beams.

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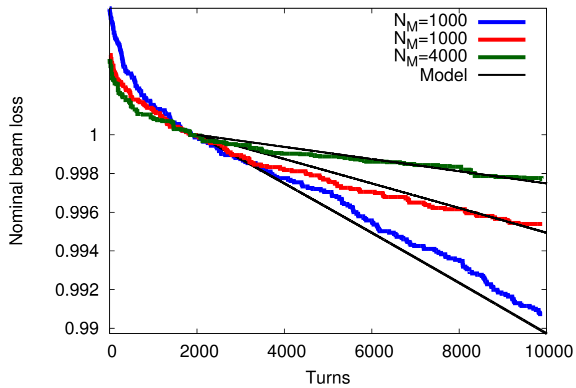

Figure 4: Beam loss pattern within the first 10,000 turns in a SIS100 scenario for a Gaussian beam with different number of macro-particles.

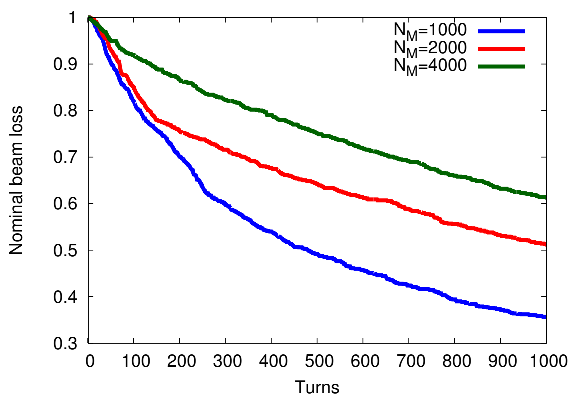

Figure 5: Beam loss pattern within the first 1000 turns in a SIS100 scenario for a K-V beam for different number of macro-particles.

The beam loss pattern for K-V beams shows an above-average beam loss during the very first turns. This is due to the fact, that the emittances of many particles are very close to the edge of acceptance, and even a small random fluctuation in the coordinate may induce beam loss. Later, the beam loss becomes more moderate, but is still much stronger compared to Gaussian beams.

SUMMARY

We developed a theory for estimating optimal simulation parameters for a maximum acceptable beam loss. The theory is based on previous investigation on the propagation and generation of numerical noise. We show with numerical simulation typical loss patterns for Gaussian and K-V beams, that are in agreement with analytical derivations.

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