

# EMITTANCE RECONSTRUCTION TECHNIQUES IN PRESENCE OF SPACE CHARGE APPLIED DURING THE LINAC4 BEAM COMMISSIONING

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## Abstract

The classical emittance reconstruction technique, based on analytic calculations using transfer matrices and beam profile measurements, is reliable only if the emittance is conserved and the space charge forces are negligible in the beamline between the reconstruction and measurement points. The effects of space charge forces prevent this method from giving sound results up to a relativistic beta of about 0.5 and make it inapplicable to the Linac4 commissioning at 50 and 100 MeV. To compensate for this drawback we have developed a dedicated technique, the forward method, which extends the classical method by combining it with an iterative process of multiparticle tracking including space charge forces. The forward method, complemented with a tomographic reconstruction routine, has been applied to transverse and longitudinal emittance reconstruction during the Linac4 beam commissioning. In this paper we describe the reconstruction process and its application during Linac4 beam commissioning.

## INTRODUCTION

The characterization of the beam emittance is essential during each commissioning stage of a linac to validate the settings and correct operation of the linac part being commissioned and predict the beam behaviour and potential beam losses downstream.

At Linac4, during the commissioning of the low energy part, a slit-grid emittance meter installed on a movable diagnostic bench [1] was used for the direct measurement of the transverse phase spaces. The bench was consecutively used for the beam measurements at 3MeV and 12MeV [2]. The longitudinal beam profiles were measured using a bunch shape monitor (BSM) [3] and longitudinal emittance was reconstructed from the measured profiles using forward method [4, 5].

At higher beam energies, the technical realization of a slit becomes challenging. Therefore, indirect emittance measurement methods based on reconstructing the emittance from profile measurements are preferred. The effects of space charge forces prevent the classical emittance reconstruction methods which depend on analytical calculations from giving sound results. Therefore, two methods, namely forward method and hybrid phase space tomography, were developed, validated and applied to the Linac4 commissioning for the emittance reconstruction in the presence of space charge. For the Linac4 50MeV and 100MeV commissioning stages, the diagnostic bench shown in Fig. 1 has been used. Transverse emittances were reconstructed at the entrance of the bench from the

measured profiles at three locations along the bench with secondary electron emission (SEM) grids. The quadrupole magnets at the entrance of the bench are essential for obtaining optimum beam sizes at the SEM grids for the reconstruction. The longitudinal emittance was reconstructed at the entrance of a cavity upstream of the bench by varying the cavity settings and measuring the longitudinal bunch profile with the BSM shown in Fig. 1.

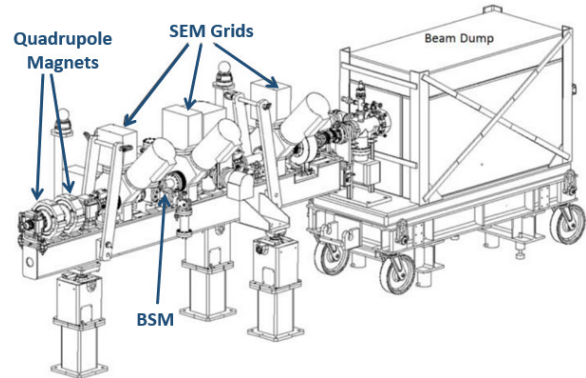


Figure 1: The Linac4 high energy diagnostic bench.

The 50MeV commissioning stage was completed in 2015 and the 100MeV commissioning stage was completed recently [6]. During each stage, the forward method and tomographic reconstruction were used to construct an estimate of the particle distribution in both transverse and longitudinal phase spaces.

The followings sections summarize the forward method with relevant references, discuss the tomographic reconstruction in detail and present the results of their applications to the measurement data.

## EMITTANCE RECONSTRUCTION IN STRONG SPACE CHARGE REGIME

The classical emittance reconstruction techniques, based on analytic calculations using transfer matrices and beam profile measurements are not reliable in strong space charge regime. Therefore, two methods are developed and applied during the Linac4 commissioning to estimate the emittance of the beam in the presence of space charge.

### The Forward Method

The forward method is a technique which aims at reconstructing the emittance of a particle beam from profile measurements in the presence of space charge. It was validated theoretically [7, 8] and experimentally [5] during the Linac4 commissioning.

The method is based on iteratively varying the Twiss parameters of the beam and tracking it to the measurement locations by including space-charge effects and comparing the measured and simulated rms beam sizes. The process continues until the simulated beam sizes converge to the measured ones.

During the Linac4 50 and 100 MeV beam commissioning, the forward method was applied to the transverse and longitudinal emittance reconstruction. The details and results will be given in the next sections.

The forward method is relatively simple, though it was proved to be powerful. At the same time, using only the rms beam size calculated from the measured profiles puts limit on the detail that can be recovered from the beam profile measurements. A more sophisticated method, phase tomography, can be used for the reconstruction of phase space density to make use of all the possible information that can be gathered from the profile measurements.

### Hybrid Phase Space Tomography

In beam physics, phase space tomography refers to the process of calculating the distribution of particle density in 2-D phase space from its 1-D projections obtained from the beam profile measurements. Tomography has been used in the field of accelerators for several decades for the reconstruction of the transverse and longitudinal phase space density [9-14]. In most of the cases, it was applied to the low current or high energy beams where the nonlinear effects like space-charge can be ignored.

The standard phase space tomography is based on linear mapping of several measured beam profiles onto the initial phase space to estimate the distribution of particle density. The measured profiles are mapped onto the phase space at the reconstruction place and the initial estimate of the distribution of particle density is obtained. The projections of the reconstructed phase space on the measured data are calculated and compared with the measured profiles. The reconstructed distribution of density in the phase space is modified iteratively until the projections of the reconstructed phase space agree with the measured profiles.

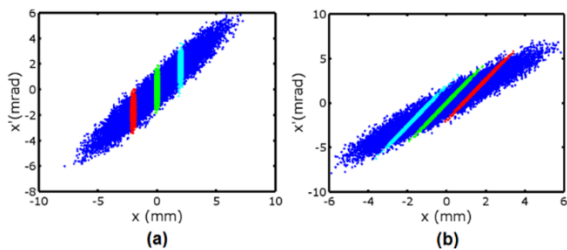


Figure 2: Horizontal phase space (a) at the third SEM grid with rays of particles (around -1, 0 and 1mm), (b) initial phase space at entrance of the diagnostics bench with the initial coordinates of the rays (no space-charge).

Figure 2a shows the horizontal phase spaces of the beam at the third SEM grid of the diagnostic bench (Fig.

1) after particle tracking without space-charge starting from the entrance of the bench. Selected rays of particles can be imagined as the particles hitting to the SEM grid wires having horizontal position of -1, 0 and 1mm. Figure 2b shows the initial phase space and the coordinates of the particles forming the rays in Fig. 2a which is identical to mapping of the coordinate of the wires onto the initial phase space. For the linear mapping of the wire positions onto the initial phase space, the transfer matrix from the reconstruction place to the profile monitor is necessary and sufficient.

When the space-charge effects are not negligible, linear mapping of the measured beam profiles onto the initial phase space is not possible. Figure 3a shows the phase space of the beam at the third SEM grid after tracking with the space-charge effects included. Figure 3b shows the initial phase space and the coordinates of the particles forming the rays in Fig. 3a. As it can be seen from the plots, when the space-charge is included, the mapping is no longer linear and the wire positions at the measurement place does not represent a line in the initial phase space anymore.

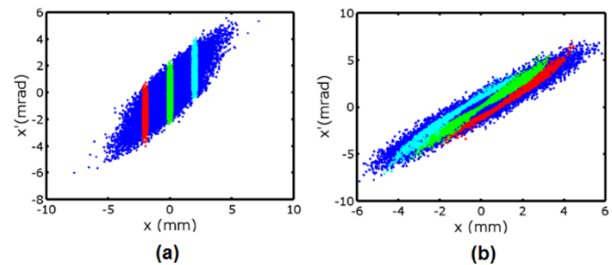


Figure 3: Horizontal phase space (a) at the third SEM grid with rays of particles (around -1, 0 and 1mm), (b) initial phase space at the entrance of the diagnostics bench with the initial coordinates of the rays (with space-charge).

In the longitudinal plane, the motion is nonlinear even if the space charge forces are ignored. Figure 4a shows the longitudinal phase space of a test beam after tracking through the Linac4 third drift tube linac (DTL) tank (Fig. 7) without space-charge. Figure 4b shows the longitudinal phase space at the entrance of the DTL tank and the initial coordinates of the particles forming the rays shown in Fig. 4a. The plots in Fig. 4 shows that even if the space-charge is neglected, the rays of particles having the same phase do not transform linearly onto the initial phase space.

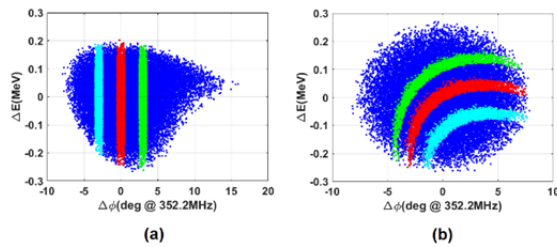


Figure 4: Longitudinal phase space (a) at the DTL exit with rays of particles (around -3, 0 and 3deg), (b) initial phase space at the entrance of the third DTL tank with the initial coordinates of the rays (without space-charge).

A hybrid phase space tomography technique which combines tomography with the multi-particle tracking including space-charge was developed, validated and applied during the Linac4 commissioning to the reconstruction of the transverse and longitudinal phase space density.

The method is iterative and based on tracking a test beam from the reconstruction place to the measurement locations and comparing the simulated beam profiles with the measured ones and modifying the initial particle distribution accordingly.

A script was written in Octave [15] by interfacing it with Travel [16] to automatize the iterative process. The multi-particle tracking is done by Travel, and the data analysis is done by Octave.

In Travel, each particle is tagged and the tag is not changed as the particles are tracked. Therefore, anywhere along the beam line, it is possible to find the coordinates of each particle in the initial phase space. This feature simplifies the process of mapping any area in the final phase space onto the initial phase space even if the transformation is not linear (see Fig. 2, 3 and 4).

The detailed process for the transverse phase space tomography at the entrance of the diagnostic bench (Fig. 1) using the measured beam profiles is as follows:

- The binned data of beam profile measurements are given as an input the script.
- A multi-particle beam (test beam) with uniform distribution and large emittance in both transverse phase spaces is generated at the entrance of the bench. For the longitudinal distribution, the distribution predicted by particle tracking upstream of the linac is used.
- The test beam is tracked to the SEM grid locations with the optics settings that were used while taking the measurements.
- The measured profiles are normalized and compared to the beam profiles obtained from the simulations.
- Each bin in the measured profiles is visited and the simulated particles falling on the bin are selected. Each particle is given a weight according to the measured signal at this bin and the number of particles falling into it. Particles falling outside of the measured profiles are given zero weight.
- At the end of the comparison, each particle gets three weights (one from each SEM grid) for each plane. The weights are combined independently in horizon-

tal and vertical planes by summing three weights if all the weights are nonzero, else the combined weight is set to zero.

- At this point, the distribution of new particle density in each transverse phase space is defined and the information is carried in the initial coordinates and the weight of each particle for the corresponding plane.
- In order to generate a new particle distribution for the next iteration, each phase space is divided into rectangular cells (cell number is defined by the user).
- Using rotation matrix, the coordinates of the particles are transformed so that the major axis of the rms ellipse in each phase space is aligned with the horizontal axis. This decreases the computation time, increases the resolution and avoids generation of phase space distributions with sharp edges.
- Each cell is visited and the total weight in this cell (relative particle density) is calculated by summing all the weights of the particles inside the cell.
- Comparing the density in each cell, the new number of particles inside the cell is computed. If necessary, the number of particles inside each cell is modified by adding or removing particles randomly. The total number of particles in the beam is kept constant in each iteration.
- After the particle generation is complete, the coordinates of the particles are transformed back to obtain the initial ellipse orientation.
- The new particle file is generated and the new iteration is started.
- At each iteration, the measured profiles and the simulated profiles (after normalizing) from the reconstructed beam are compared. For each bin of the measured profile, absolute difference of the measured and simulated signal is calculated. The integral of this function (discrepancy) gives an idea about the convergence (Fig. 6).

The accuracy of the method was tested theoretically at 50 MeV with an H<sup>+</sup> beam of 20 mA. A reference beam at the entrance of the diagnostic bench was simulated to the SEM grids and the beam profiles are obtained. A test beam was generated with uniform particle distribution in the transverse phase spaces and a longitudinal distribution identical to that of the reference beam. Using the procedure explained above, the horizontal and vertical phase spaces were reconstructed from the beam profiles. Figure 5 shows the phase space plots of the reference and the reconstructed beams.

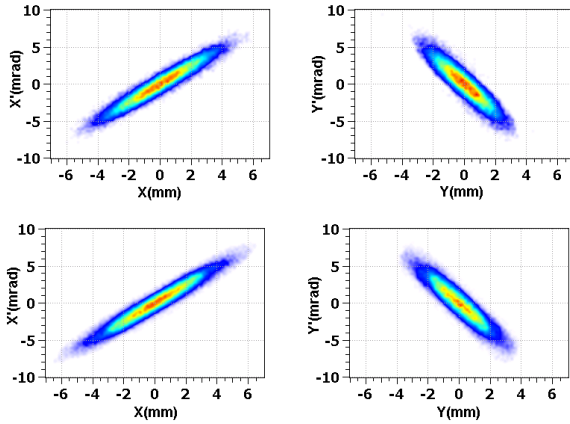


Figure 5: Transverse phase space plots of the reference beam (top row) and the reconstructed beam (bottom row).

The Twiss parameters of the reference and reconstructed beams are given in Table 1. In this specific test, the reconstruction method was able to predict the rms emittances with an error less than 3% and the other Twiss parameters less than 10%. The accuracy of the reconstruction may be increased by introducing more profiles for the reconstruction.

Figure 6 shows the evolution of discrepancy between the measured and simulated profiles after each iteration. Experience with the method showed that the variation of the curves are similar for different reconstructions. After about eight iterations, the method converges to a solution and during the next iterations the variation in the phase spaces is very small.

Table 1: Twiss Parameters (Emittance is Normalized) of the Reference and Reconstructed Beams at 50 MeV with 20 mA Beam Current

Parameter	Reference	Reconstructed
$\epsilon_x$ (rms)	$0.39 \pi$ .mm.mrad	$0.40 \pi$ .mm.mrad
$\alpha_x$	-3.47	-3.81
$\beta_x$	$2.86 \text{ mm}/\pi$ .mrad	$3.14 \text{ mm}/\pi$ .mrad
$\epsilon_y$ (rms)	$0.35 \pi$ .mm.mrad	$0.36 \pi$ .mm.mrad
$\alpha_y$	2.35	2.49
$\beta_y$	$1.28 \text{ mm}/\pi$ .mrad	$1.39 \text{ mm}/\pi$ .mrad

In order to have accurate calculation of the space-charge effects, it is important to use a realistic longitudinal distribution for the initial beam at the first iteration. For this specific test, two transverse phase spaces were reconstructed simultaneously. However, it is possible to reconstruct only one transverse phase space at a time. In that case, the accurate description of the other transverse phase space and longitudinal phase space is important for the accuracy of the reconstruction.

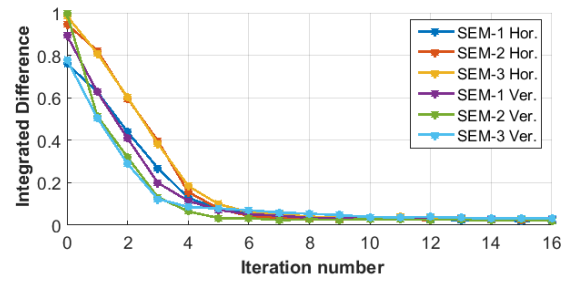


Figure 6: Discrepancy between the measured and simulated profiles after each iteration.

In the case of Linac4 transverse emittance measurements, the profiles were taken at three locations along the beam line. However the method can also be applied to different cases. For instance, when a focusing element is varied and the profiles are measured at the same location. Likewise, it can be applied to the longitudinal phase space where the phase and/or amplitude of an RF cavity is varied and the phase or momentum profile is measured downstream. The former is the basis of the longitudinal emittance measurements during the Linac4 commissioning.

A similar method which uses turn-by-turn particle tracking is being used at CERN for the longitudinal emittance measurements in the PS Booster [17, 18].

### 50 MEV COMMISSIONING STAGE

During the 50 MeV beam commissioning stage, the diagnostic bench shown in Fig. 1 was installed downstream of the third DTL tank (Fig. 7) which accelerates the beam from 30MeV to 50MeV. The forward method and tomographic reconstruction was applied to the measurement data at 50MeV H<sup>+</sup> beam with 20 mA beam current.



Figure 7: A sketch of part of Linac4 during 50 MeV commissioning.

#### Transverse Emittance Measurements

The forward method and the hybrid phase space tomography were applied to a set of horizontal and vertical beam profiles measured along the diagnostic bench for the transverse emittance measurements. Figure 8 shows the reconstructed transverse phase spaces with the forward method (top row) and phase space tomography (bottom row).



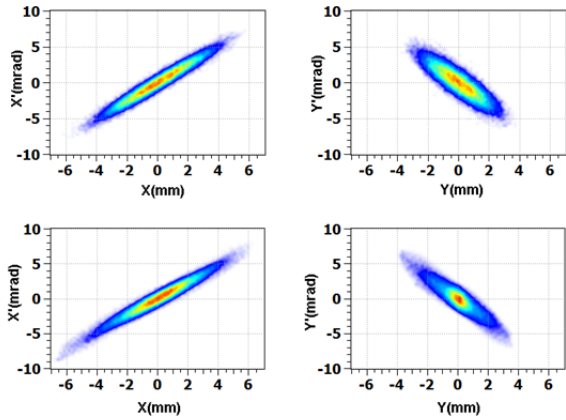


Figure 8: Reconstructed transverse phase spaces. Top row: forward method, bottom row: tomographic reconstruction.

The reconstructed rms emittances are  $\epsilon_x = 0.32 \pi \cdot \text{mm} \cdot \text{mrad}$  and  $\epsilon_y = 0.36 \pi \cdot \text{mm} \cdot \text{mrad}$  from the forward method and  $\epsilon_x = 0.33 \pi \cdot \text{mm} \cdot \text{mrad}$  and  $\epsilon_y = 0.32 \pi \cdot \text{mm} \cdot \text{mrad}$  from the phase space tomography. The other Twiss parameters in each plane have less than 10% difference. The particle distribution reconstructed with the tomographic method has differences compared to what was initially assumed for the forward method.

### Longitudinal Emittance Measurements

The longitudinal phase space density of the beam was reconstructed at the entrance of the DTL tank-3 (Fig. 7) from the longitudinal bunch profiles. The profiles were measured with the BSM on the diagnostic bench by varying the RF phase of the tank. The forward method and the hybrid phase space tomography were applied to the same measurement data. The reconstructed longitudinal phase spaces are given in Fig. 9. The reconstructed rms emittances are  $0.29 \pi \cdot \text{deg} \cdot \text{MeV}$  and  $0.33 \pi \cdot \text{deg} \cdot \text{MeV}$  with forward method and phase space tomography respectively.

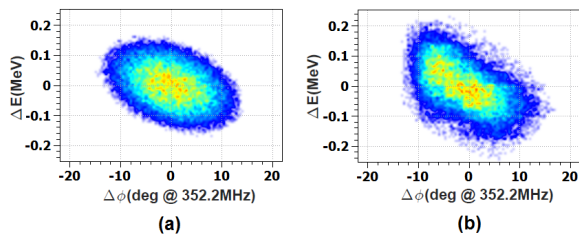


Figure 9: Reconstructed longitudinal phase space, (a) forward method and (b) tomographic reconstruction.

### 100 MEV COMMISSIONING STAGE

During the 100 MeV beam commissioning, the high energy diagnostic bench was connected to the first tank of the Pi-Mode Structure (PIMS), as shown in Fig. 10.



Figure 10: A sketch of part of Linac4 during 100 MeV commissioning.

Parallel to the process of setting the cavities [6], the transverse phase spaces of the beam were reconstructed at the entrance of the diagnostic bench (Fig. 10) at 50 MeV and 80 MeV beam energies. During the measurements at 50 MeV, all the cavities after the DTL tank-3 were turned off. Likewise, during the measurements at 80 MeV, all the cavities after the fourth module of the Cell-Coupled Drift Tube Linac (CCDTL) were turned off. The Twiss parameters of the reconstructed beams from the forward method and tomographic reconstruction differ less than 10 % at each beam energy.

In order to compare the beams reconstructed with phase space tomography, the reconstructed beam at 50 MeV was backtracked [19] to the exit of the DTL tank-3 with the CCDTL and PIMS cavities turned off and then tracked forward to the reconstruction place with the first four CCDTL cavities are on. The distance between the DTL tank-3 exit and the diagnostic bench was around 27 m.

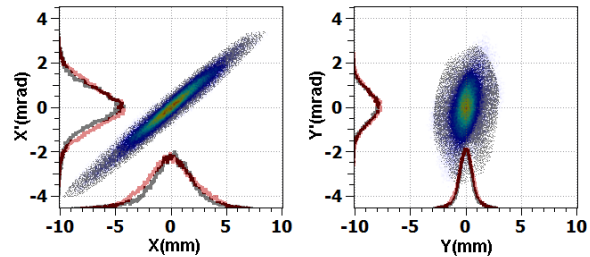


Figure 11: Comparison of phase space plots of the beams measured at 80MeV (grayscale) and measured at 50 MeV then tracked to 80MeV (colour scale).

In Fig. 11, the transverse phase space plots of the beam measured at 80 MeV are overlapped with the phase space plots of the beam measured at 50 MeV and tracked to 80MeV. As it can be seen from the figure, the particle distribution obtained from two sets of measurements at different energies and brought to the same conditions by simulations agree very well. The twiss parameters of the two beams are compared in Table. 2. All the parameters, except  $\alpha_y$ , are very close. The big percentage difference in  $\alpha_y$  may come from the fact that it is close to zero.

Table 2: Twiss Parameters (Emittance is Normalized) of the Beams Measured at 80MeV and Measured at 50 MeV then Tracked to 80MeV

Parameter	50 MeV tracked to 80 MeV	Measured at 80 MeV
$\epsilon_x$ (rms)	0.22 $\pi$ .mm.mrad	0.23 $\pi$ .mm.mrad
$\alpha_x$	-4.5	-4.6
$\beta_x$	10.7 mm/ $\pi$ .mrad	10.6 mm/ $\pi$ .mrad
$\epsilon_y$ (rms)	0.26 $\pi$ .mm.mrad	0.27 $\pi$ .mm.mrad
$\alpha_y$	-0.49	-0.19
$\beta_y$	1.03 mm/ $\pi$ .mrad	1.02 mm/ $\pi$ .mrad

The agreement of the phase space plots given in Fig. 11 and the parameters given in Table 2 confirms the correct operation of the focusing channel along the beamline where the 50 MeV measured beam was first backtracked and then forward tracked.

At the 100 MeV commissioning stage, the measured emittance of the beam in transverse planes is smaller than what was measured at 50 MeV stage. This is mostly due to the fact that the profile measurements were taken when the transmission through the linac was not optimized.

## CONCLUSION

The “forward method” and the “hybrid phase space tomography” were developed, validated and successfully applied during the Linac4 commissioning. These methods allow measurement of the emittance based on the observation of transverse/longitudinal profiles also under the effect of space charge. Both methods give sound and consistent results with each other for the prediction of the rms ellipse parameters. Moreover, the hybrid phase space tomography allows reconstruction of the distribution of particle density in a phase space from the measured beam profiles. These methods, which can be applied to any linac during commissioning and operation, were essential for the Linac4 50 MeV and 100 MeV beam commissioning. The comparison of the measured emittance, at different locations along the linac, with the expected values enabled validation of the operational machine settings and the reconstructed beam distribution was instrumental for the preparation of the next commissioning stages. Both methods will be used during the beam commissioning at 160 MeV and permanently during the operation of Linac4 at the end of the linac, as well as at the PS Booster injection.

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