Head-Tail Modes With Strong Space Charge: Theory and Simulations

Alexey Burov

Fermilab

Many thanks to Alex Macridin and Tim Zolkin

TMCI Suppression

PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS, VOLUME **1**, 044201 (1998)

Fast head-tail instability with space charge

M. Blaskiewicz
Alternating Gradient Synchrotron Department, Brookhaven National Laboratory, Upton, New York 11973-5000
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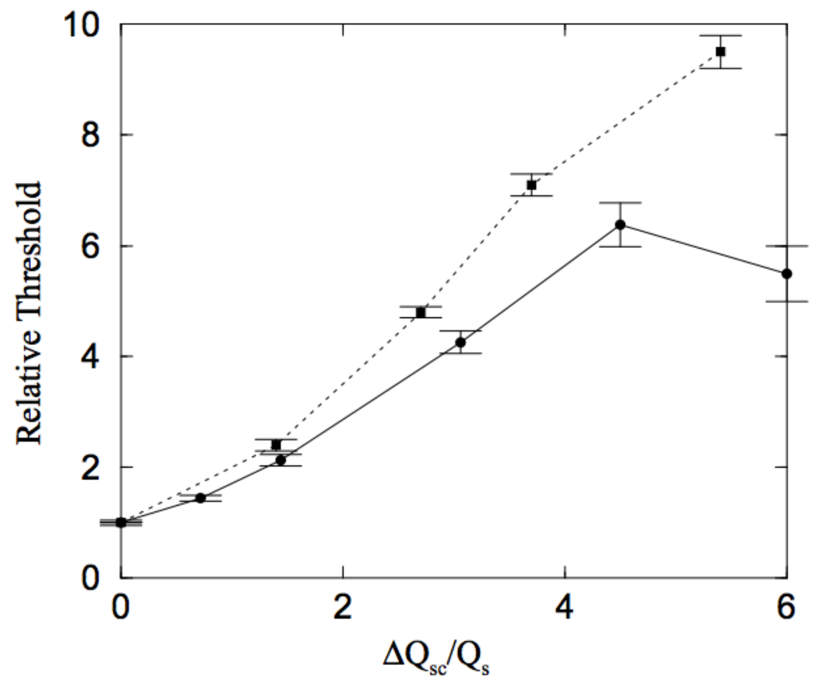

FIG. 9. Threshold wake in units of the threshold wake for $\Delta Q_{sc} = 0$ vs $\Delta Q_{sc}/Q_s$ for different line densities: $\mu = 1$, circles, solid line; $\mu = -1/2$, squares, short dashed line.

Stability Issues of Low-Energy Intense Beams

Vanishing TMCI

Burov & Ng, 1999

FERMILAB-FN-685

October 1999

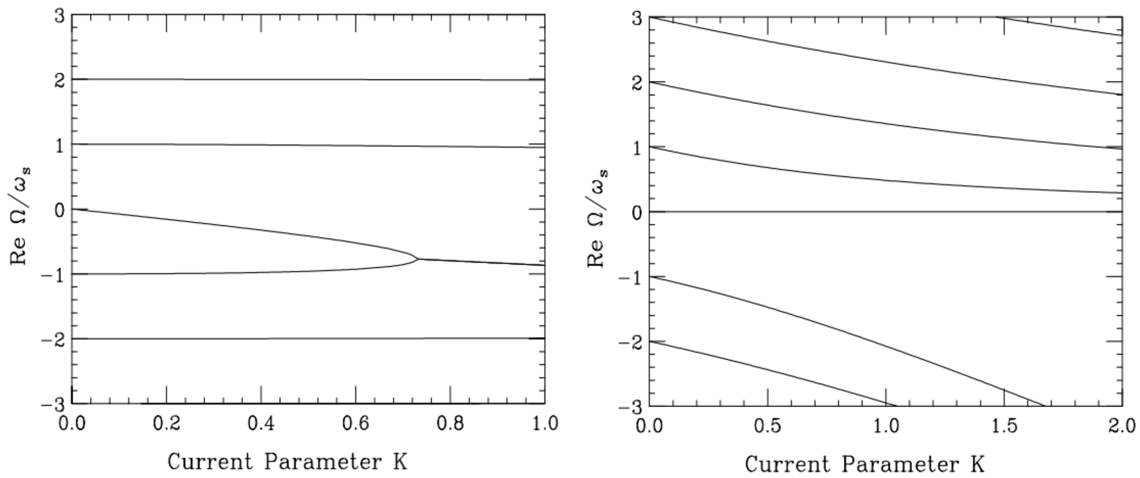

FIGURE 8. Left: The transverse wake force shifts mostly the azimuthal 0 mode downward but not the other modes. Instability occurs when the 0 and -1 modes meet with each other. Right: The space-charge force in the absence of the wake forces shifts all modes downward with the exception of the 0 mode.

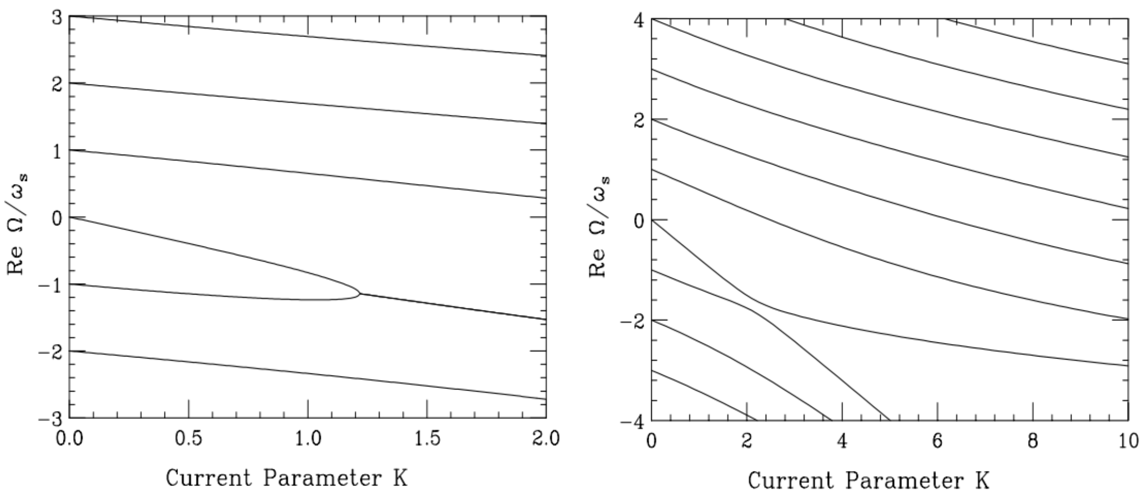

FIGURE 9. Left: With the transverse space-charge force added to the wake forces, all modes except the 0 mode are shifted downward, thus requiring the 0 and -1 modes to couple at a much higher current threshold. Right: When space charge reaches the critical value of $\xi = 5$, the -1 mode is shifted away from the 0 mode by so much that they do not couple anymore.

Strong Space Charge Approximation:

$q = \Delta Q_{sc} / Q_s \gg 2k+1$ $\quad$ q is the space charge parameter

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Head-tail modes for strong space charge

A. Burov

FNAL, Batavia, Illinois 60510, USA

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Arbitrary bunch distribution function, both longitudinal and transverse

Arbitrary RF shape

Arbitrary number of bunches, train structure, dampers

Arbitrary driving and detuning wakes.

SSC+Wakes, no LD

$$\nu\bar{y}(\tau) + \frac{1}{Q_{\text{eff}}(\tau)}\frac{d}{d\tau}\left(u^2\frac{d\bar{y}}{d\tau}\right) = \kappa(\hat{\mathbf{W}}\,\bar{y} + \hat{\mathbf{D}}\,\bar{y})$$

$$\bar{y}'(\pm\infty) = 0$$

$$\kappa = \frac{r_0 R}{4\pi\beta^2\gamma Q_b}$$

$$Q_{\text{eff}}(\tau) = 0.52 Q_{\max}\exp\left(-\tau^2/2\right)$$

$$\hat{\mathbf{W}}\,\bar{y} \equiv \int_\tau^\infty W(\tau - s)\exp[i\zeta(\tau - s)]\rho(s)\bar{y}(s)ds$$

$$\hat{\mathbf{D}}\,\bar{y} \equiv \bar{y}(\tau)\int_\tau^\infty D(\tau - s)\rho(s)ds$$

SSC+Wakes, no LD

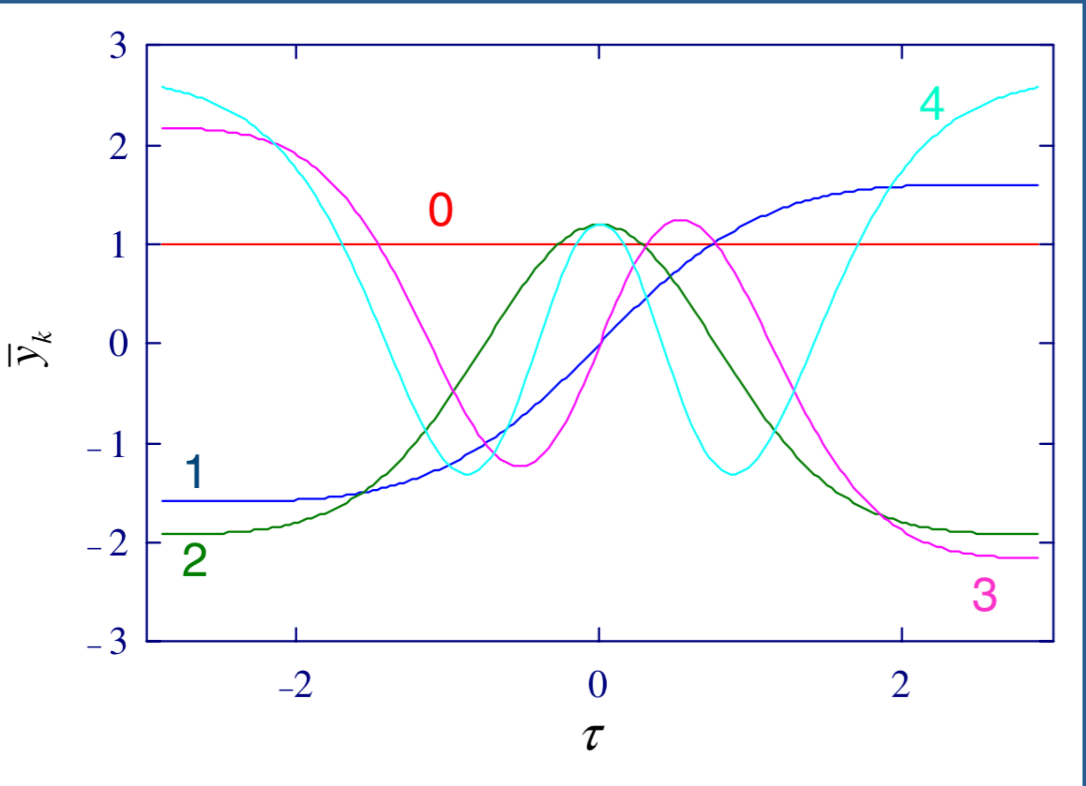
With $$\bar{y}(\tau) = \sum_{k=0}^{\infty} B_k \bar{y}_{0k}(\tau),$$:

$$[\kappa \widehat{\mathbf{W}} + \kappa \widehat{\mathbf{D}} + \mathrm{Diag}(\boldsymbol{\nu}_0)]\mathbf{B} = \nu \mathbf{B}.$$

$$\hat{\mathbf{W}}_{km} = N_b^{-1} \int_{-\infty}^{\infty} \int_{\tau}^{\infty} W(\tau - s) \exp[i\zeta(\tau - s)] \rho(s) \rho(\tau) \bar{y}_{0k}(\tau) \bar{y}_{0m}(s) ds d\tau,$$

$$\hat{\mathbf{D}}_{km} = N_b^{-1} \int_{-\infty}^{\infty} \int_{\tau}^{\infty} D(\tau - s) \rho(s) \rho(\tau) \bar{y}_{0k}(\tau) \bar{y}_{0m}(\tau) ds d\tau.$$

No-Wake Eigen-system: 1D array of natural numbers



$$\bar{y}'' + \nu \exp(-\tau^2/2)\bar{y} = 0,$$

$$\bar{y}'(\pm\infty) = 0$$

the coherent tune shift ν is in units

$$Q_s^2 / \Delta Q_{\text{eff}} \equiv Q_s / q_{\text{eff}}$$

$$\Delta Q_{\text{eff}} = 0.52 \Delta Q_{\max}$$

TABLE I. Eigenvalues and asymptotic values for the first ten modes of the Gaussian bunch.

k	0	1	2	3	4	5	6	7	8	9
ν	0	1.4	4.4	8.9	15	23	32	43	56	70
$\|y_\infty\|$	1	1.6	2.0	2.3	2.5	2.7	2.9	3.0	3.2	3.3

Non-monotonic TMCI Threshold vs SC Resistive Wall and similar wakes

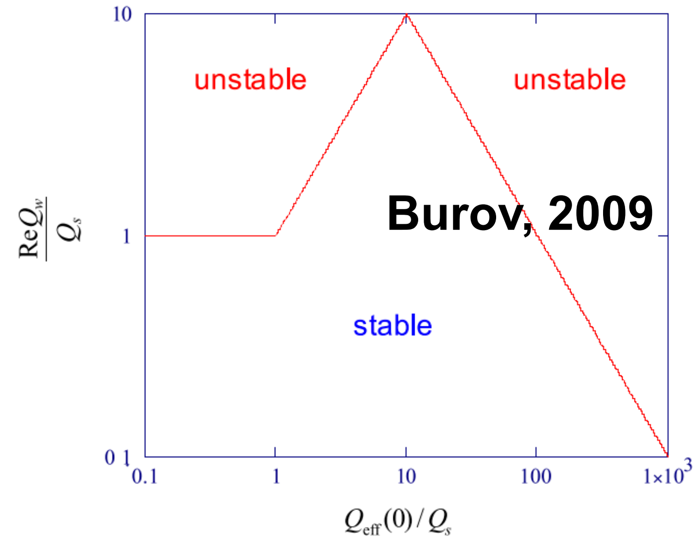
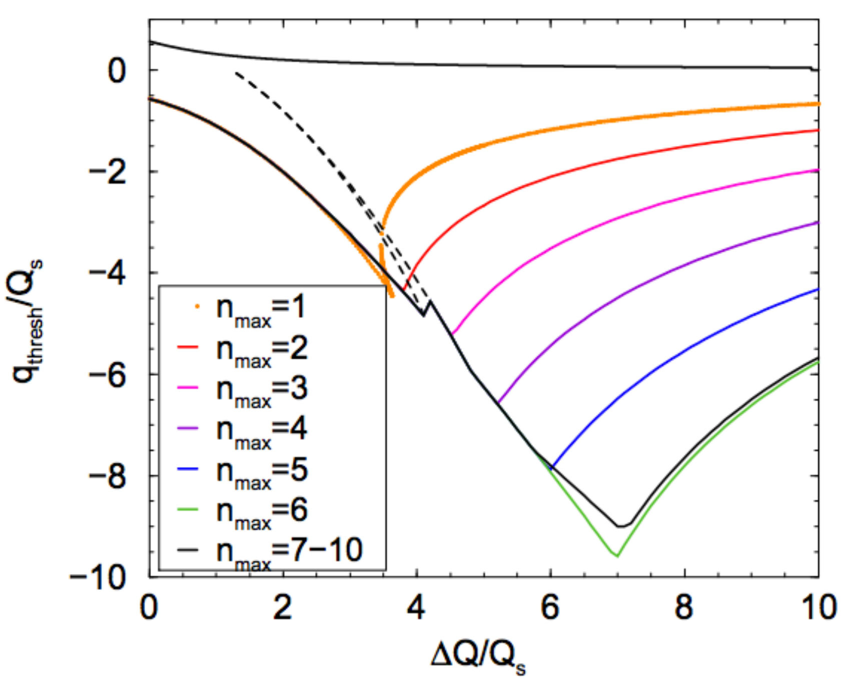
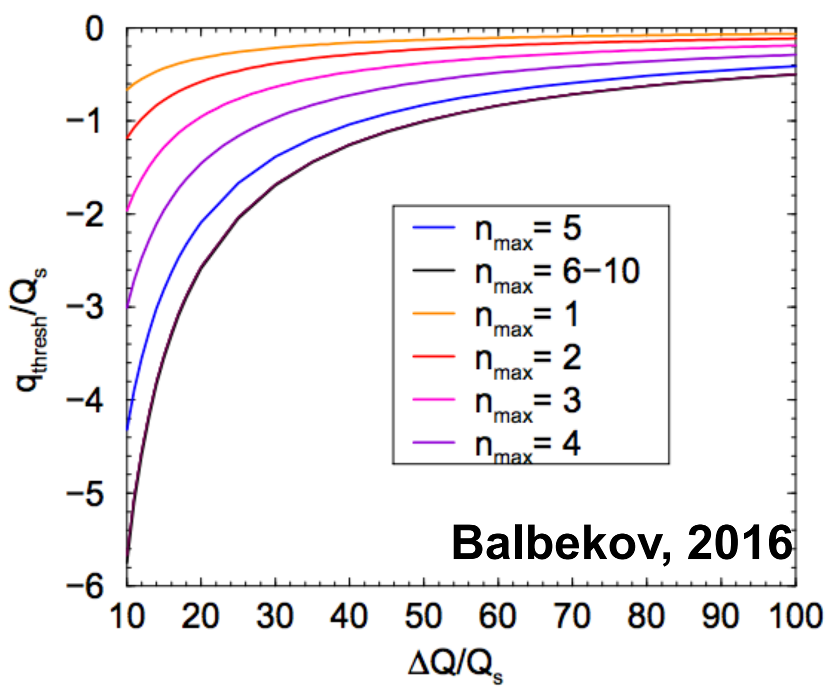


FIG. 7. (Color) A schematic behavior of the TMCI threshold for the coherent tune shift versus the space charge tune shift. Both tune shifts are in units of the synchrotron tune.





Damper

For a space-flat damper

$$\kappa \hat{\mathbf{W}} \to \kappa \hat{\mathbf{W}} + g\hat{\mathbf{G}}$$

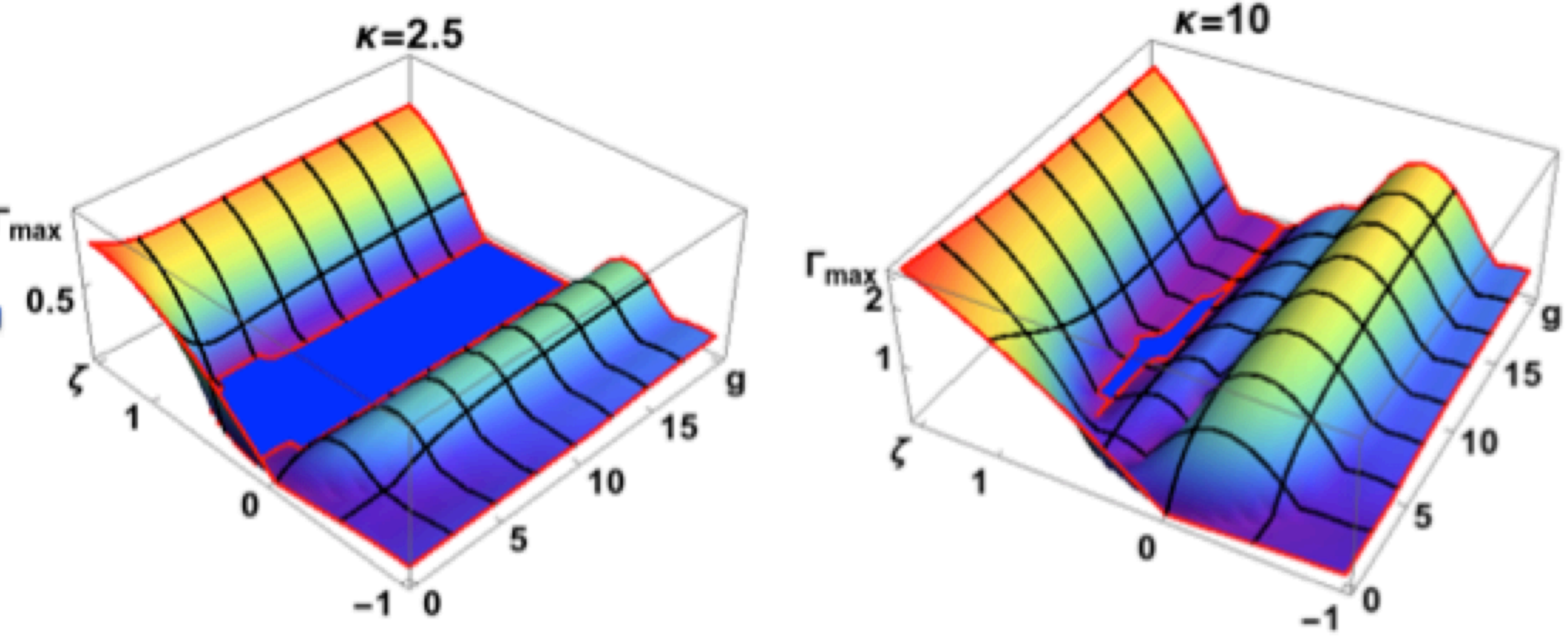
$$\hat{\mathbf{G}}_{lm} = I_l I_m^*; \quad I_l = \int_{-\infty}^{\infty} d\tau \exp(i\zeta\tau)\rho(\tau)\bar{y}_l(\tau)$$

If it is not flat, the pickup / kicker form-factors can be taken into account:

$$I_l = \int_{-\infty}^{\infty} d\tau \exp(i\zeta\tau)\rho(\tau)\bar{y}_l(\tau)K(\tau)$$

$$I_m = \int_{-\infty}^{\infty} d\tau \exp(i\zeta\tau)\rho(\tau)\bar{y}_l(\tau)P(\tau)$$

SB Wake + Damper: 2D stability area ("lake")



Kappa is the intensity parameter. For RR, kappa=3.5

Courtesy of Tim Zolkin

Coupled Bunches, Flat CB Wake

$$\kappa \hat{\mathbf{W}} \to \kappa \hat{\mathbf{W}} + \kappa \hat{\mathbf{G}} \tilde{W}_{\mu}$$

$$\tilde{W}_{\mu} = \sum_{k=1}^{\infty} W(-ks_0) \exp(ik\phi_{\mu});$$

$$\phi_{\mu} = 2\pi(\mu + Q_x)/M; \qquad s_0 = 2\pi R_0/M$$

More details on the dampers and coupled-bunch with SSC and ZSC

Coupled-Beam and Coupled-Bunch Instabilities

A. Burov*

Fermilab, PO Box 500, Batavia, IL 60510-5011

(Dated: June 27, 2016)

A problem of coupled-beam instability is solved for two multibunch beams with slightly different revolution frequencies, as in the Fermilab Recycler Ring (RR). Sharing of the inter-bunch growth rates between the intra-bunch modes is described. The general analysis is applied to the RR; possibilities to stabilize the beams by means of chromaticity, feedback and Landau damping are considered.

http://arxiv.org/pdf/1606.07430v1.pdf

Landau Damping, analytical

Intrinsic LD:

$$\Lambda_k = 0.5\,\frac{\nu_k^2 y_{\infty k}^2}{\tau_*}\,\frac{Q_s}{q^3} \sim k^4\,\frac{Q_s}{q^3}$$

Octupole's LD:

$$\delta Q_{\mathrm{oct}} > 0; \quad \Lambda_k \sim 30\,\frac{\delta Q_{\mathrm{oct}}^2}{\Delta Q_{\max}} \ll \frac{(k+1)Q_s}{q}$$

The numerical factors have to be checked

The scaling has to be checked

Intermediate SC is interesting as well

Landau Damping, PATRIC tracking

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Head-tail instability and Landau damping in bunches with space charge

V. Kornilov and O. Boine-Frankenheim

GSI Helmholtzzentrum fuer Schwerionenforschung GmbH, Planckstraße 1, 64291 Darmstadt, Germany

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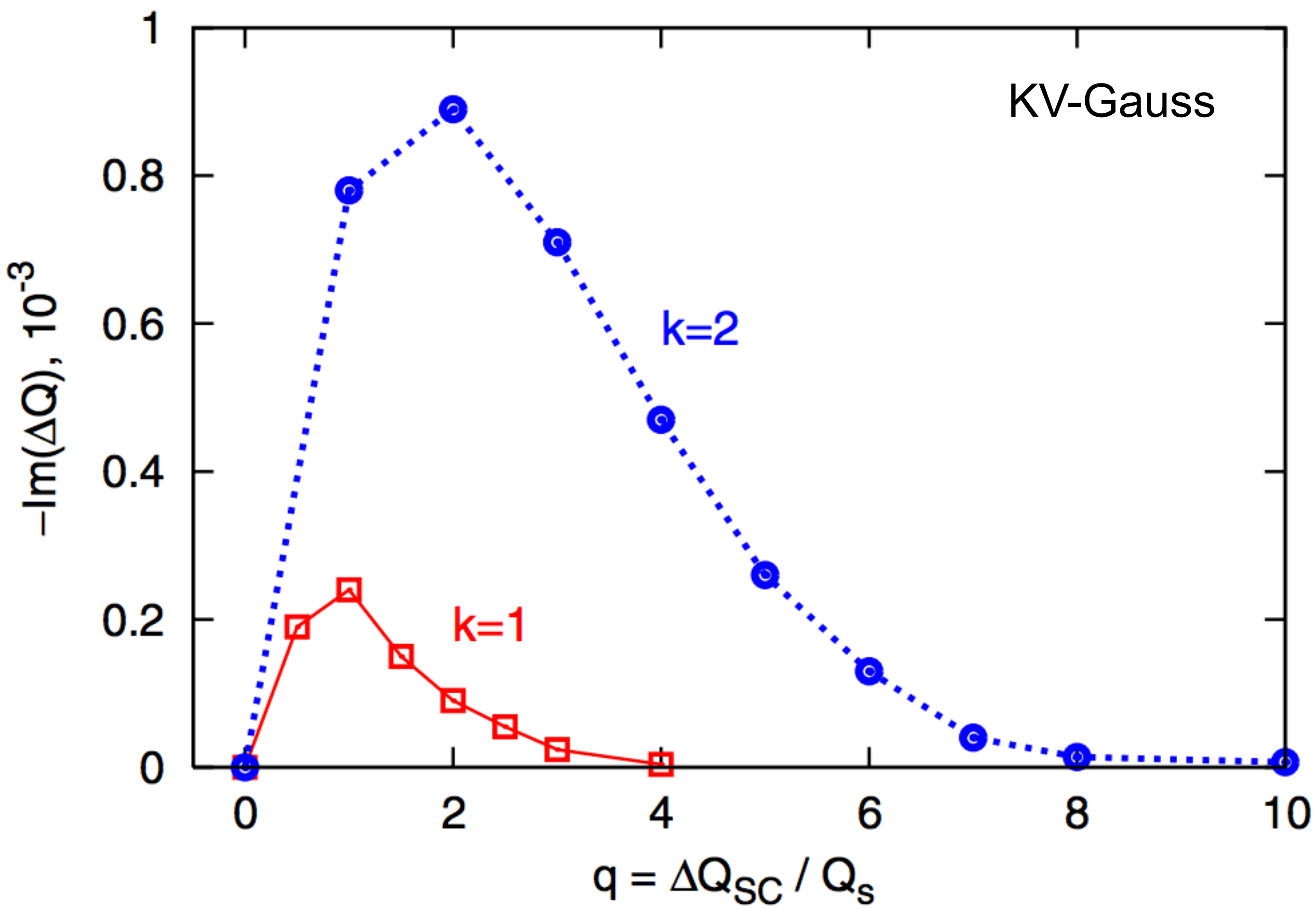
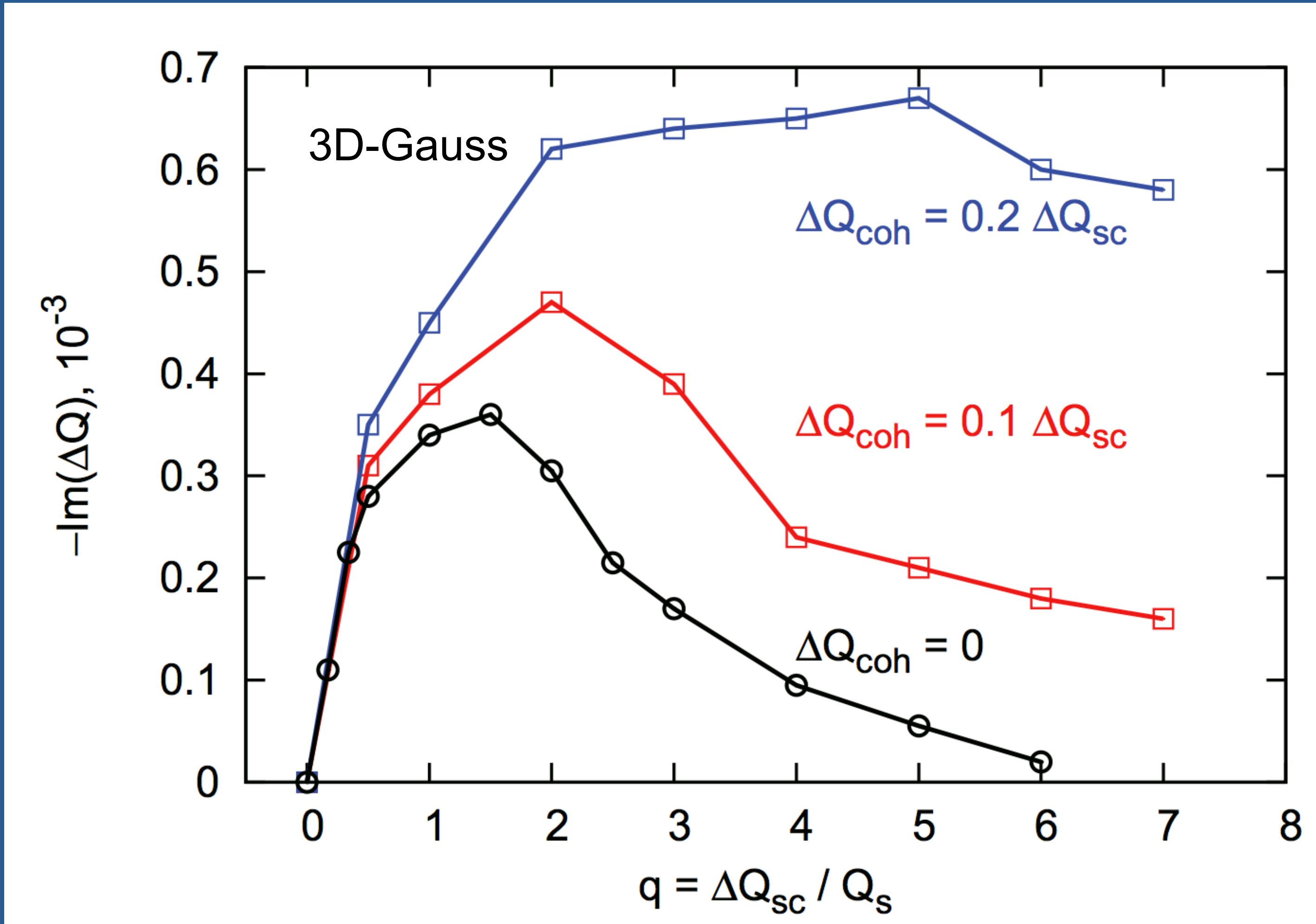


Landau Damping, PATRIC tracking

WEO1LR02 Proceedings of HB2014, East-Lansing, MI, USA

THRESHOLDS OF THE HEAD-TAIL INSTABILITY IN BUNCHES WITH SPACE CHARGE

V. Kornilov, O.Boine-Frankenheim, GSI Darmstadt, and TU Darmstadt, Germany
C. Warsop, D. Adams, B. Jones, B.G. Pine, R. Williamson, STFC/RAL/ISIS, Oxfordshire, UK



SYNERGIA Tracking vs Theory

PHYSICAL REVIEW SPECIAL TOPICS—ACCELERATORS AND BEAMS **18,** 074401 (2015)

Simulation of transverse modes with their intrinsic Landau damping for bunched beams in the presence of space charge

Alexandru Macridin, Alexey Burov, Eric Stern, James Amundson, and Panagiotis Spentzouris

Fermilab, P.O. Box 500, Batavia, Illinois 60510, USA

(Received 29 May 2015; published 22 July 2015)

Transverse dipole modes in bunches with space charge are simulated using the SYNERGIA accelerator modeling package and analyzed with dynamic mode decomposition. The properties of the first three space charge modes, including their shape, damping rates, and tune shifts are described over the entire range of space charge strength. The intrinsic Landau damping predicted and estimated in 2009 by one of the authors is confirmed with a reasonable scaling factor of $\simeq$2.4. For the KV distribution, very good agreement with PATRIC simulations performed by Kornilov and Boine-Frankenheim is obtained.

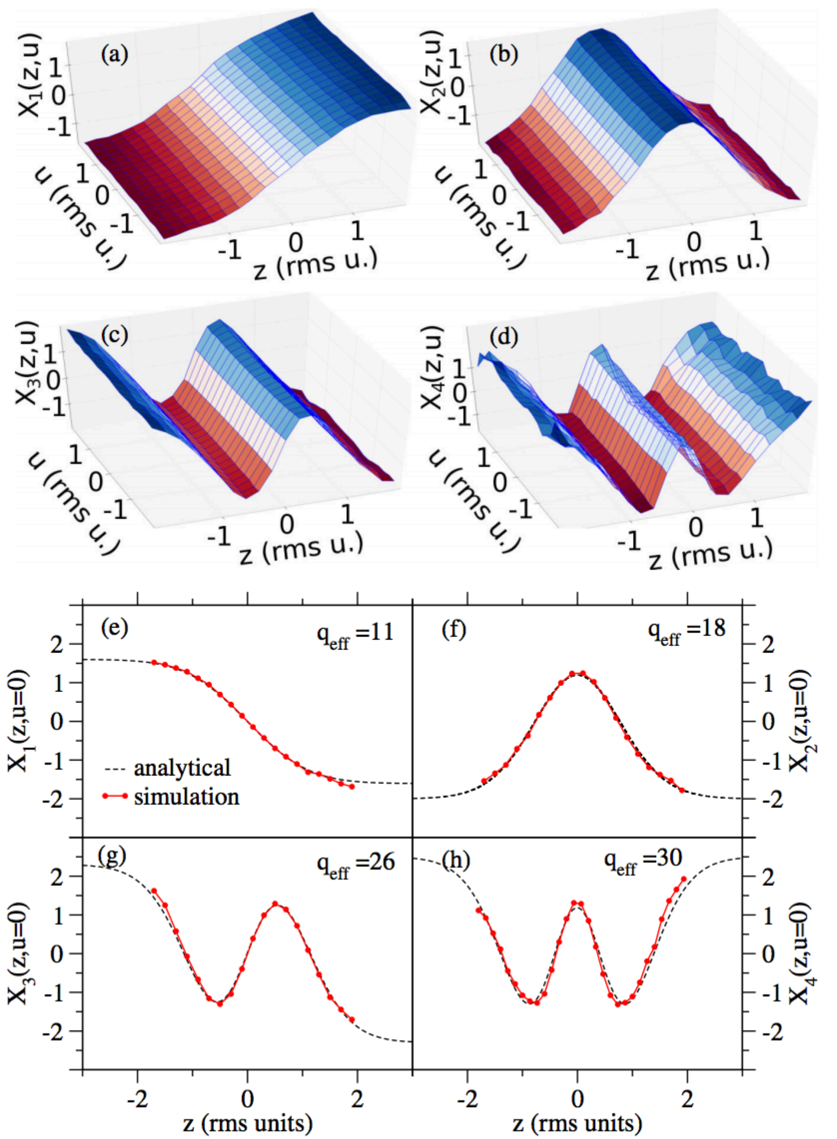


FIG. 1. The first four modes of a 3D-G bunch in the strong space charge regime. (a)–(d) At large q_{eff} the modes' spatial distribution is nearly independent of u, i.e., $X(z,u) \approx Y(z)$, as predicted by Ref. [6]. (e)–(h) Comparison of the simulated modes (solid red) with the theoretical space charge harmonics (dashed black) [6]. The agreement is very good.

Mode Shapes, 3D-Gauss

$$\mathrm{SSC:}\quad q \gg 2k$$

$$\bar{y}'' + \nu \exp(-\tau^2/2)\bar{y} = 0,$$

$$\bar{y}'(\pm\infty) = 0$$

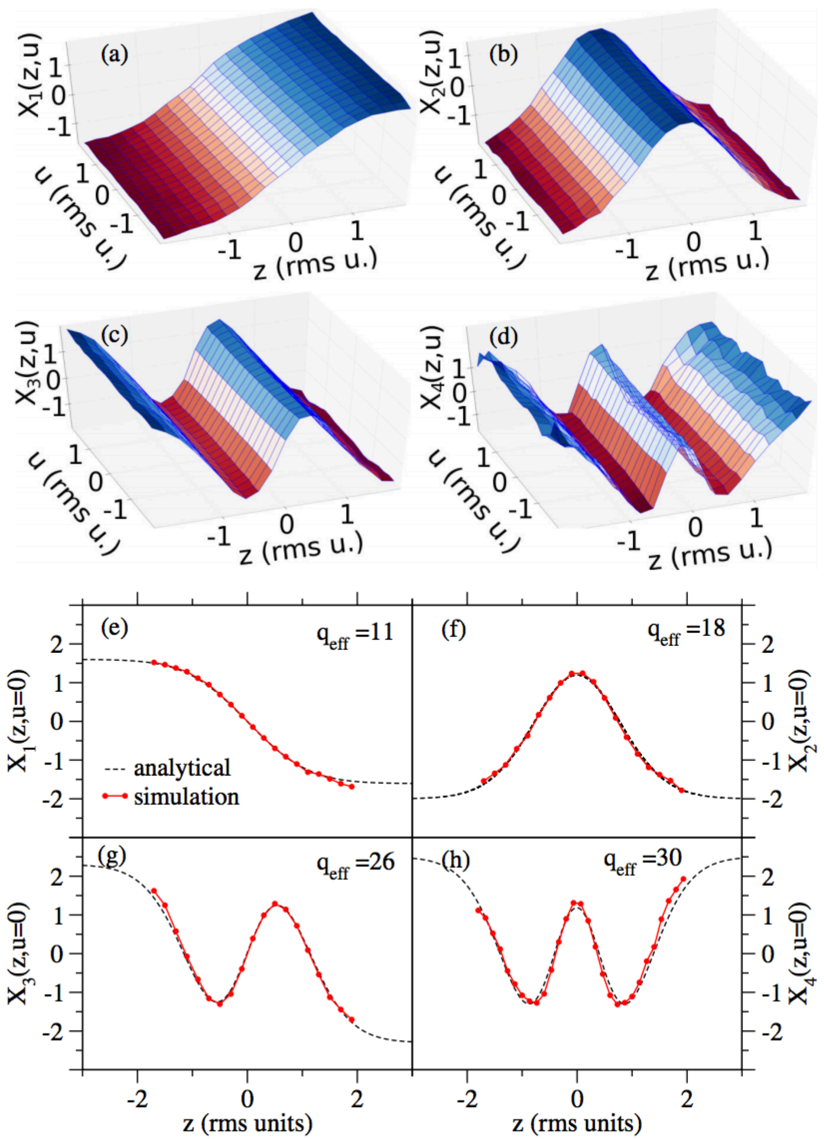
the coherent tune shift ν is in units

$$Q_s^2 / \Delta Q_{\mathrm{eff}} \equiv Q_s / q_{\mathrm{eff}}$$

$$\Delta Q_{\mathrm{eff}} = 0.52 \Delta Q_{\mathrm{max}}$$

Mode Shapes

$\mathrm{SSC:}\quad q \gg 2k$



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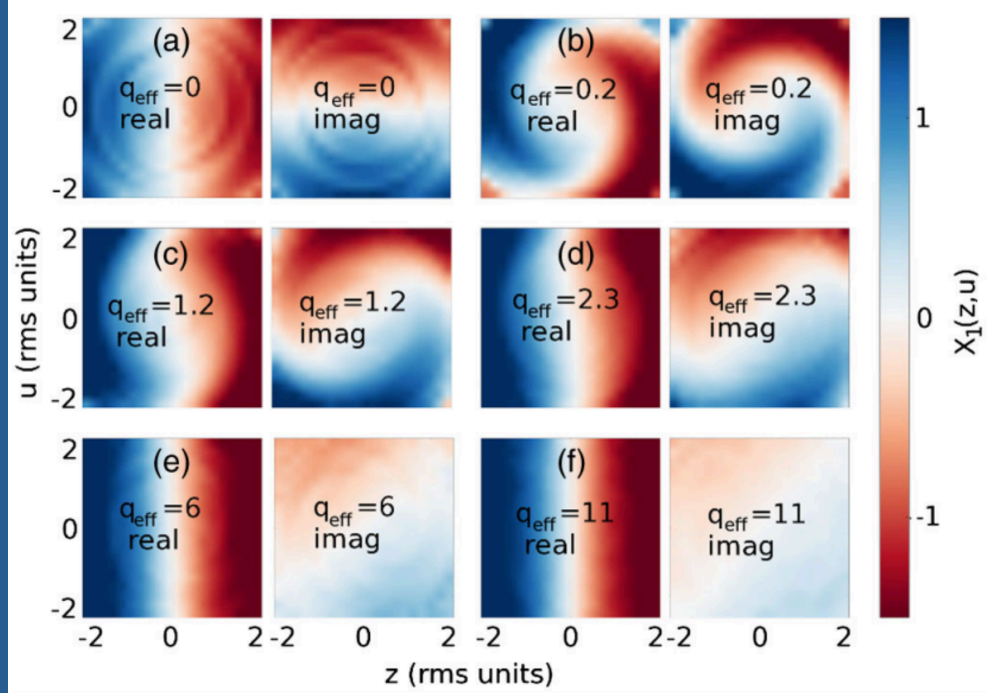
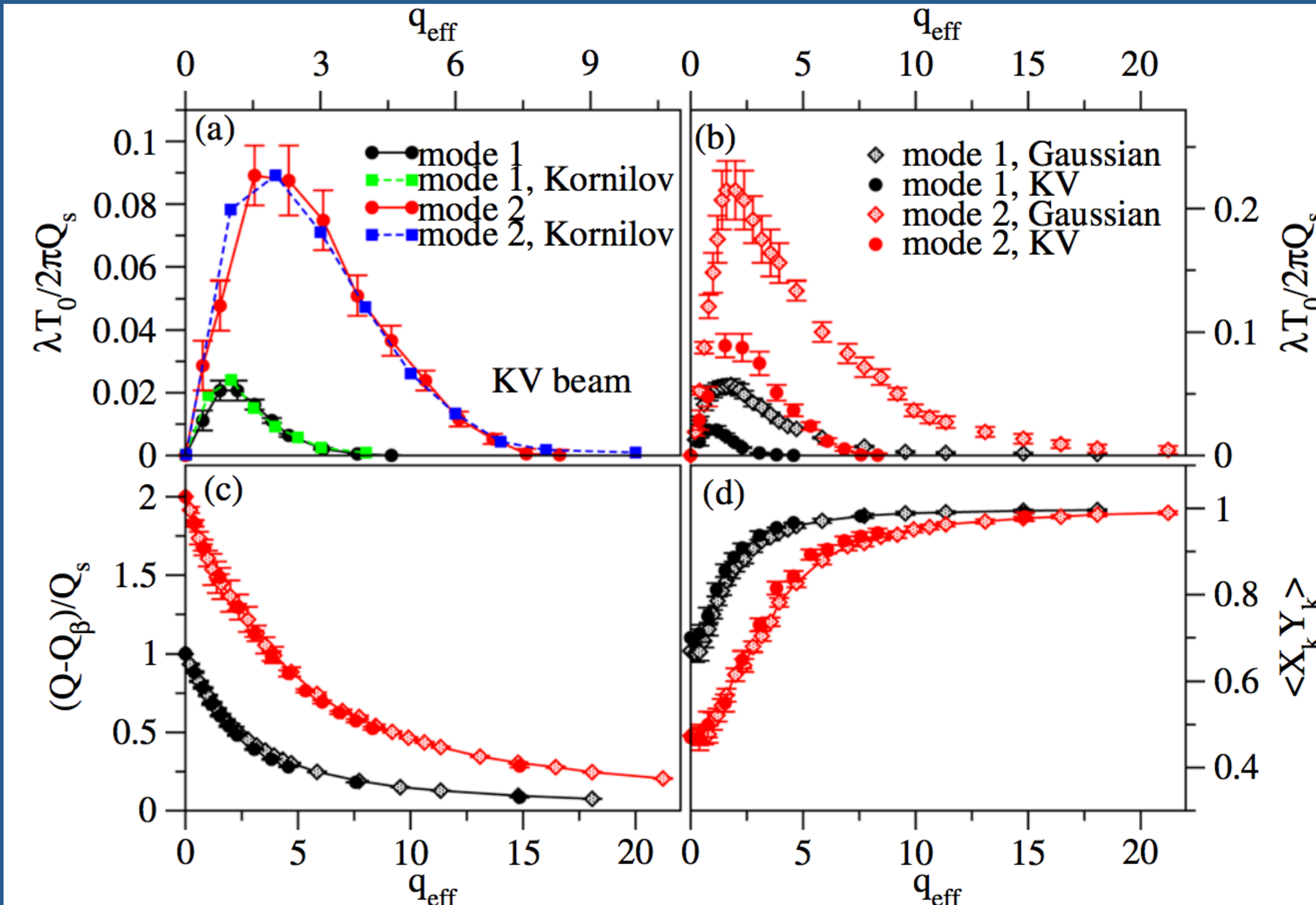


FIG. 2. 3D-G bunch. (a)–(f) The mode 1 longitudinal distribution, $X_1(z,u)$, for different values of the space charge parameter q_{eff}. Without space charge, $X_1(z,u) \propto e^{i\theta}$. With increasing q_{eff}, $X_1(z,u)$ transforms gradually to the first space charge harmonic [see Fig. 1(a)]. At large q_{eff} $X_1(z,u)$ can be described by a purely real function.

KV vs Gaussian, SYNERGIA vs PATRIC



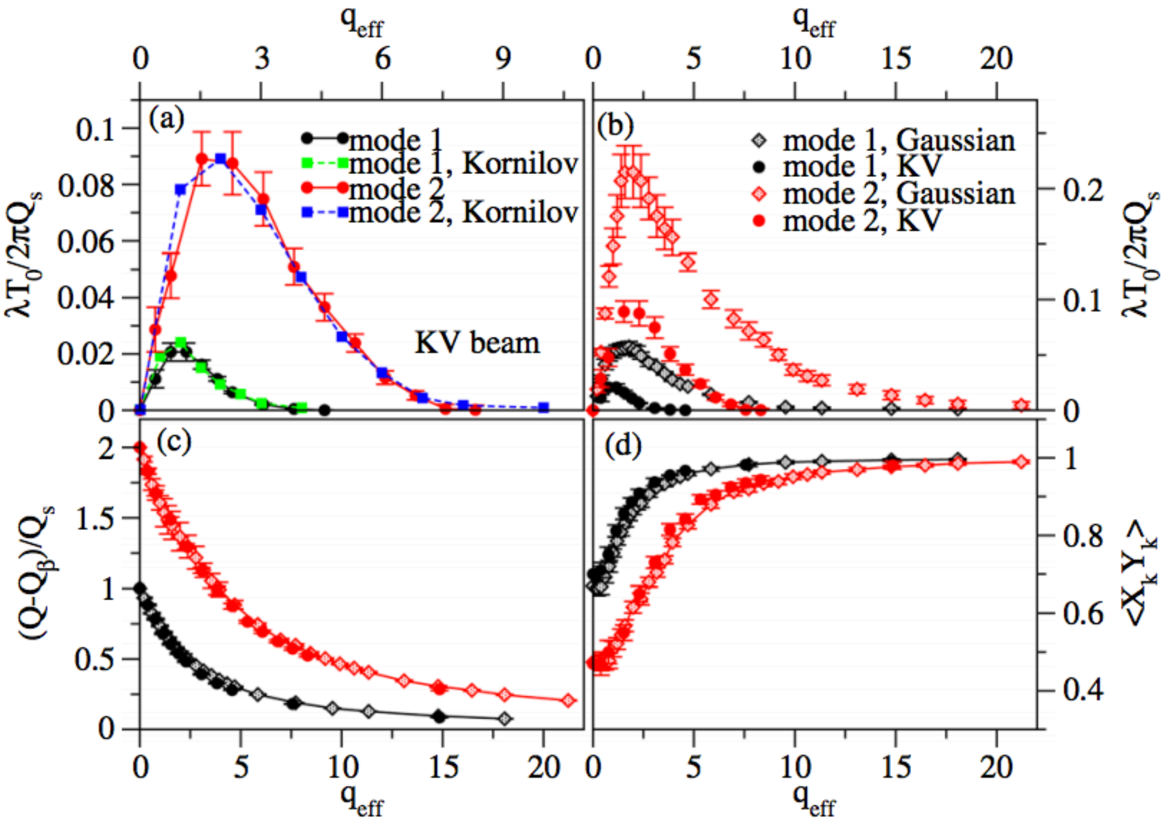


FIG. 9. (a) Landau damping for KV-G bunches. Comparison between our simulations and those of Kornilov and Boine-Frankenheim [9]. The agreement is good. (b)–(d) Comparison between the first two modes of KV-G beams and 3D-G beams. (b) Landau damping. The damping of the 3D-G beams' modes is much larger. (c) Relative tune shift. (d) Spatial overlap of the mode shape with the space charge harmonic [see Eq. (32)]. Unlike the Landau damping the tune shift and the modes shapes depend very little on the transverse beam distribution, as expected.

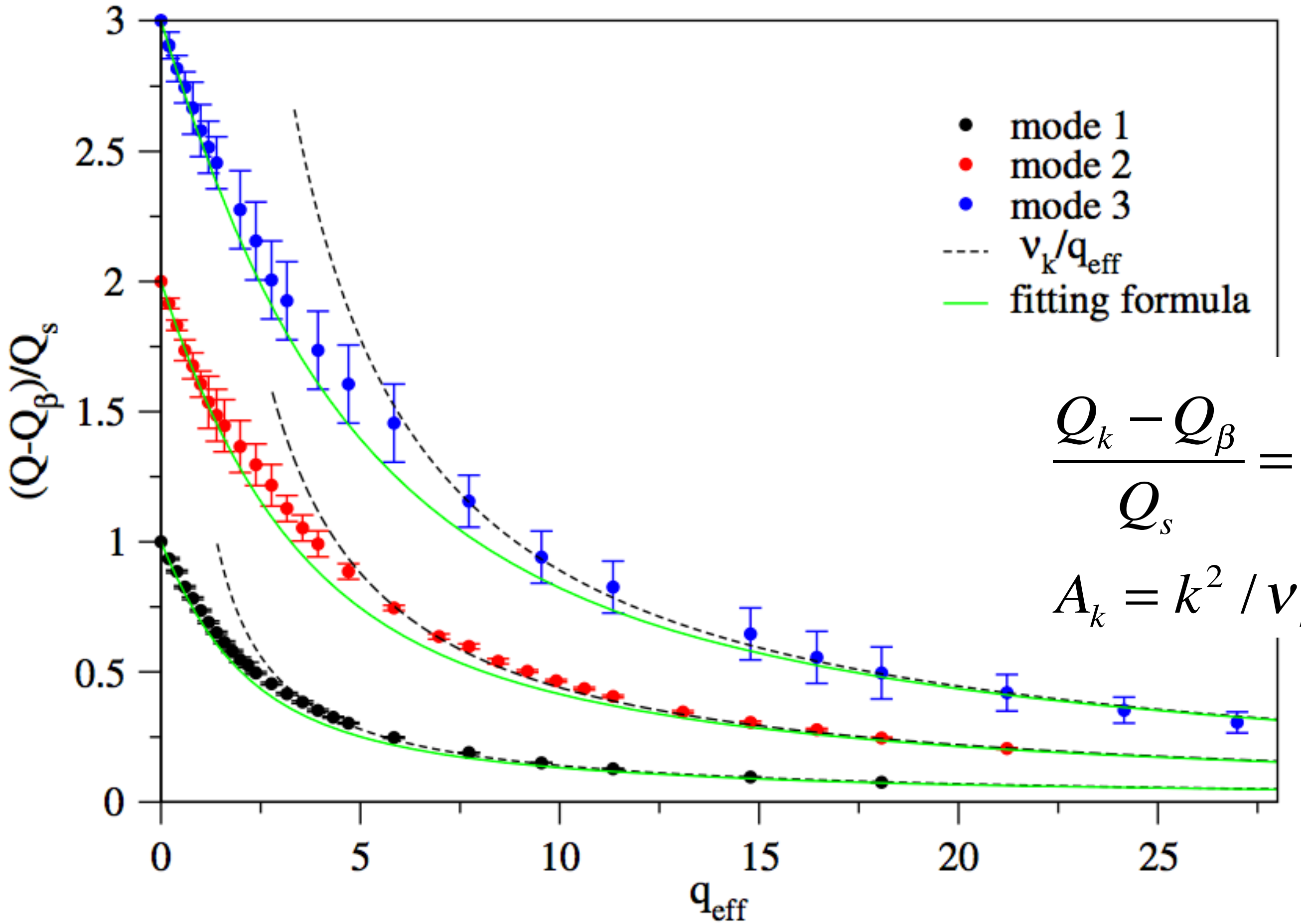
KV vs Gaussian,

SYNERGIA

vs

PATRIC

Coherent Tunes

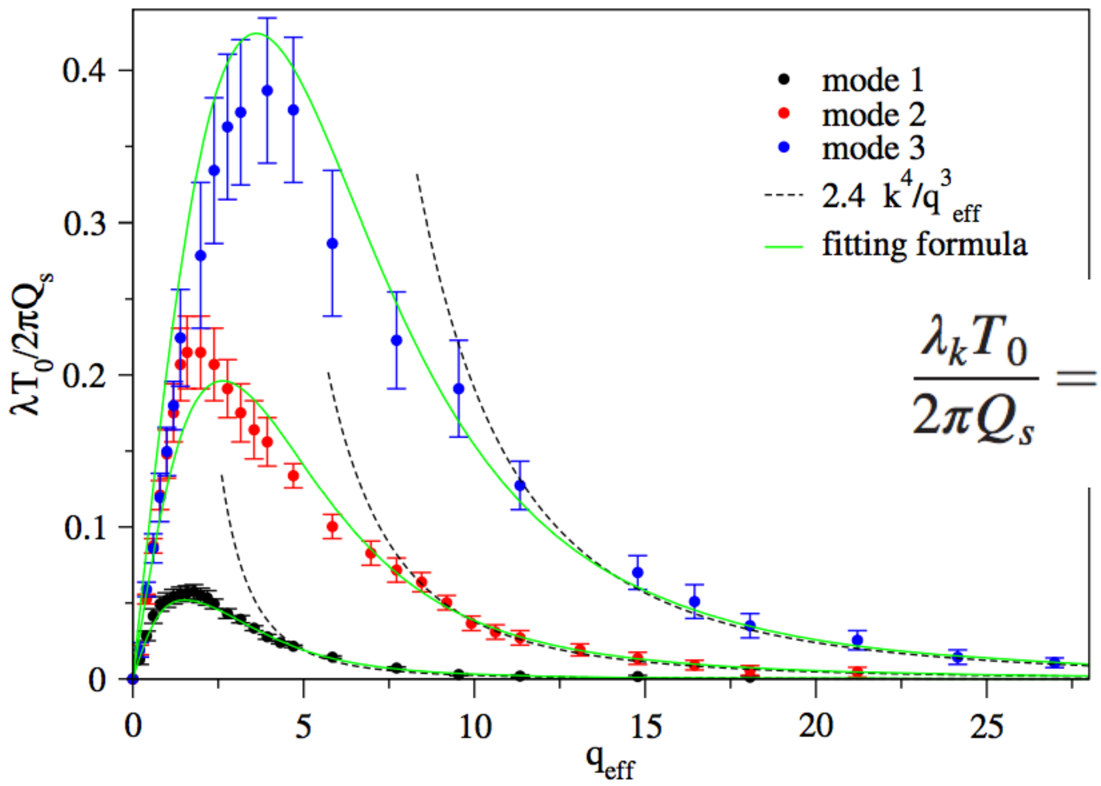


$$\frac{Q_k - Q_\beta}{Q_s} = -\frac{A_k q}{2} + \sqrt{\left(\frac{A_k q}{2}\right)^2 + k^2}$$

$$A_k = k^2 / \nu_k; \quad \nu_k \cong k^2$$

FIG. 7. 3D-G bunch. The relative tune shift, $\frac{Q-Q_\beta}{Q_s}$, for the modes 1, 2, and 3 versus the space charge parameter q_{eff}. In the strong space charge regime, $q > \approx 4k$, $\frac{\Delta Q_k}{Q_s} \approx \frac{\nu_k}{q_{\mathrm{eff}}}$, in good agreement with the theoretical prediction. $\nu_1 = 1.4$, $\nu_2 = 4.4$, and $\nu_3 = 8.9$ [6]. The tune shift can be fitted reasonably well for the entire range of the space charge strength by employing Eq. (34) (green lines).

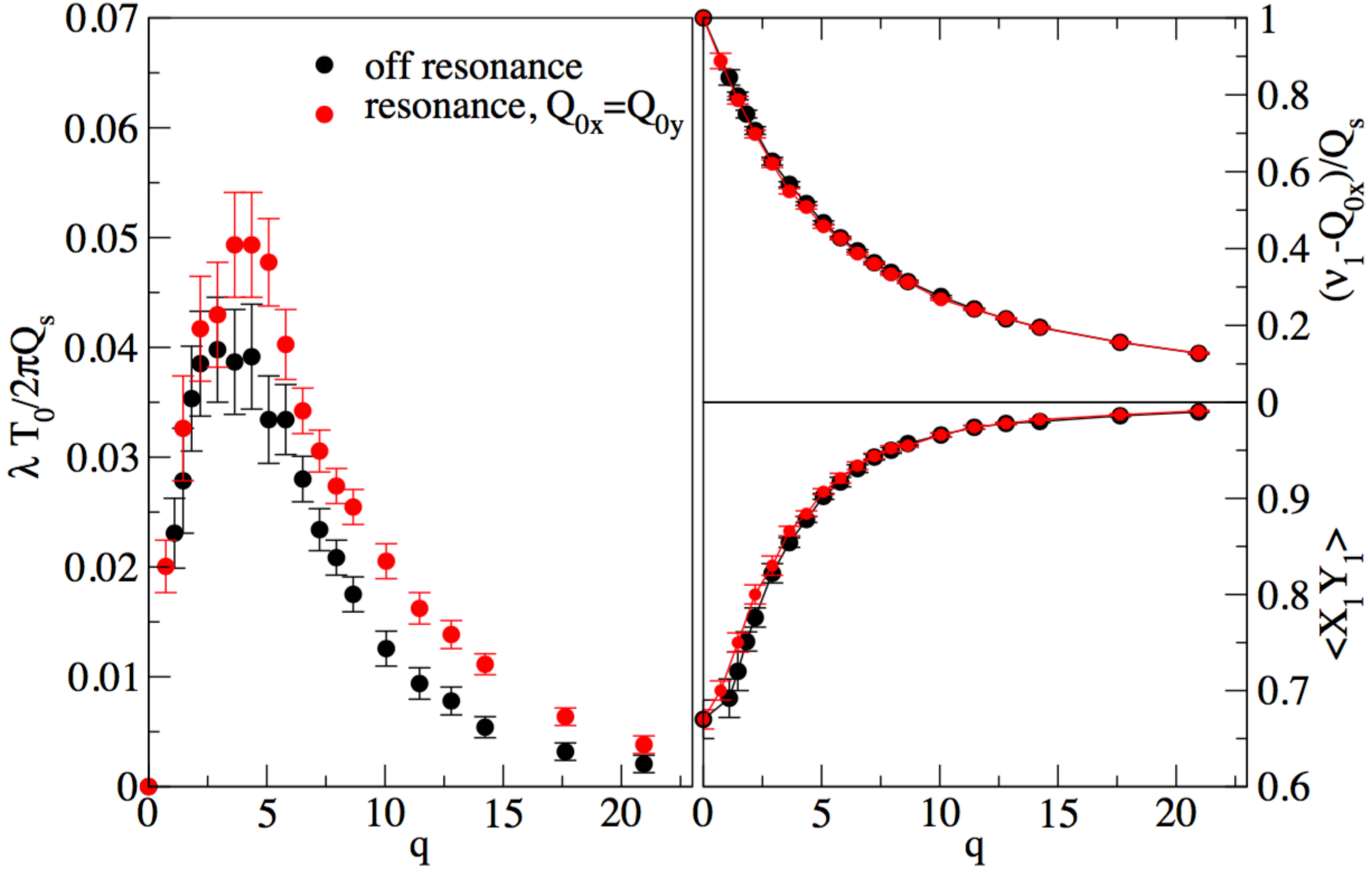
Landau Damping



$$\frac{\lambda_k T_0}{2\pi Q_s} = \frac{A k q_{\rm eff}}{1 + B(\frac{q_{\rm eff}}{2k})^2 + C(\frac{q_{\rm eff}}{2k})^3 + kD(\frac{q_{\rm eff}}{2k})^4}$$

FIG. 6. 3D-G bunch. The Landau damping for modes 1, 2, and 3 versus the space charge parameter $q_{\rm eff}$. At small $q_{\rm eff}$ the damping increases quickly with increasing $q_{\rm eff}$. In the strong space charge regime, $q > \approx 4k$, we find that $\lambda T_0 \approx 2.4 \frac{k^4 2\pi Q_s}{q^3_{\rm eff}}$, where k is the mode number (dashed lines). This behavior is in agreement with the theoretical predictions [6]. The proportionality factor of 2.4 is characteristic of transverse Gaussian beams. The damping rates of all three modes can be fitted reasonably well for the entire range of the space charge strength by employing Eq. (33) (green lines).

A. Macridin et al., 2016, to be published



Out of the coupling resonance, the agreement with my prediction is even better:

$$2.4k^4/q^3 \rightarrow (1.2 \div 1.4)k^4/q^3$$

Conclusions

- In 2009, theory of SSC was formulated for arbitrary SB and CB wakes, RF form, 3D distribution functions and feedbacks.
- For modes shapes and tunes, the problem is reduced to a standard eigensystem problem of the linear algebra.
- Mode shapes, tunes and damping rates for intrinsic and octupolar LD were quantitatively predicted.
- Many of the theoretical predictions are already confirmed in recent simulations, some other are still to be done; the work is in progress.

Many thanks!

Outline

- Particles and Waves
- SSC Approximation: the general equation
- SB wakes: weak and strong HT, TMCI thresholds.
- CB wakes and feedbacks
- Landau Damping: theory and tracking simulations
- Some Results

Particles and Waves

- In general, collective beam dynamics can be approached both in the language of particles (tracking codes) and in the language of waves (Vlasov equation, VE).

- Each of the approaches has its advantages and limitations.

- Tracking codes are applicable for any problem (in principle). However, a multi-parameter survey for multi-bunch beams still requires enormous computing resources.

- Wave approach can be many orders of magnitude faster than particle tracking. The problem is that no effective algorithm for VE is found yet for general space charge (SC).

- Problems for VE: proper basis and Landau damping

Strong Space Charge Equation, No Wake, No LD

The problem was reduced to 1D Sturm-Liouville problem:

$$\frac{d}{d\tau}\left(u^2 \frac{d\bar{y}}{d\tau}\right) + \nu Q \bar{y} = 0.$$

$$\bar{y}'(\pm\infty) = 0$$

$$u^2(\tau) \equiv \frac{\int_{-\infty}^{\infty} v^2 f(v, \tau) dv}{\int_{-\infty}^{\infty} f(v, \tau) dv}$$

$Q(\tau) \propto \lambda(\tau)$ is the local space charge tune shift, transversely averaged.

Strong Space Charge Equation, No Wake, No LD

For a Gaussian bunch

$$\bar{y}\,'' + \nu \exp(-\tau^2/2)\bar{y} = 0,$$

$$\bar{y}'(\pm\infty) = 0$$

the coherent tune shift ν is in units

$$\omega_0 Q_s^2 / \Delta Q_{sc} \equiv \omega_s / q$$

No-Wake Eigen-system: 1D array of natural numbers

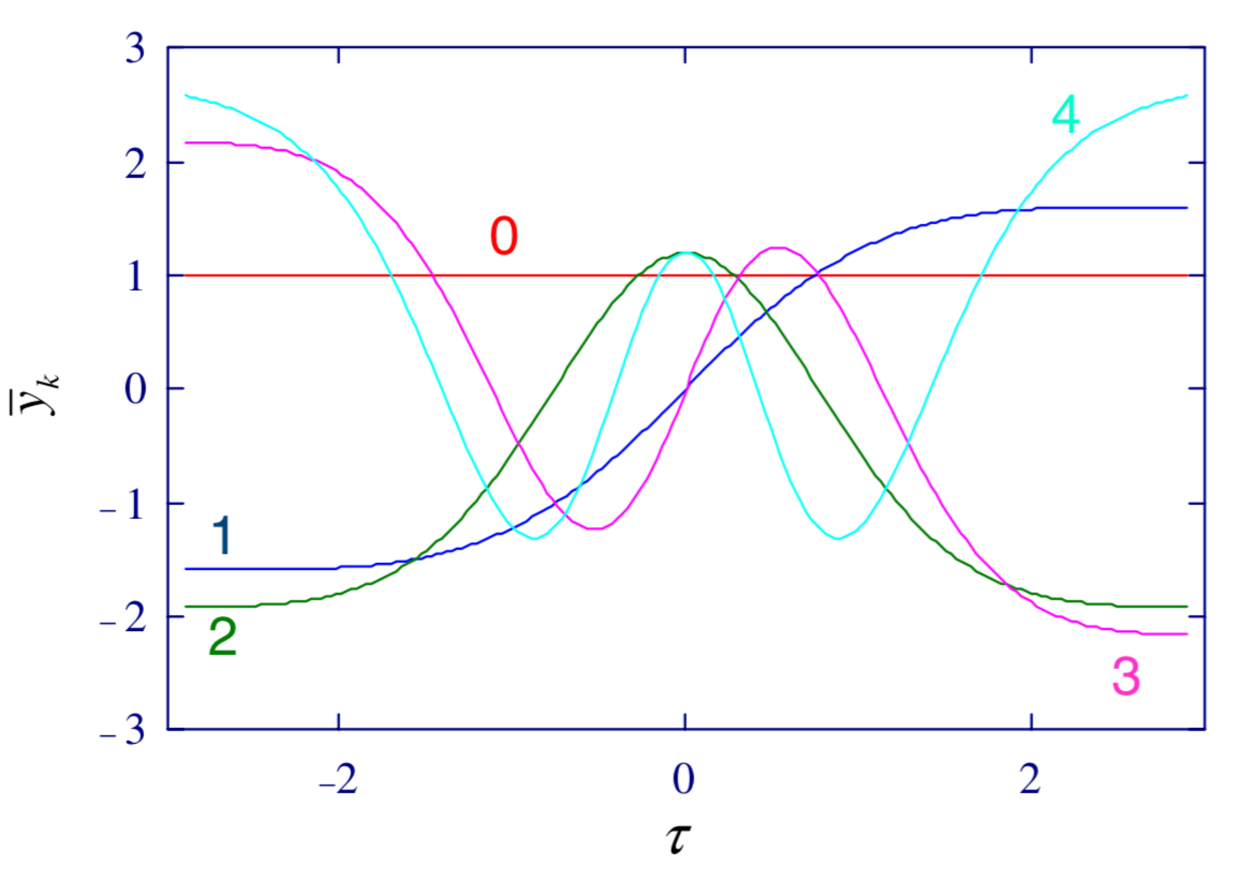
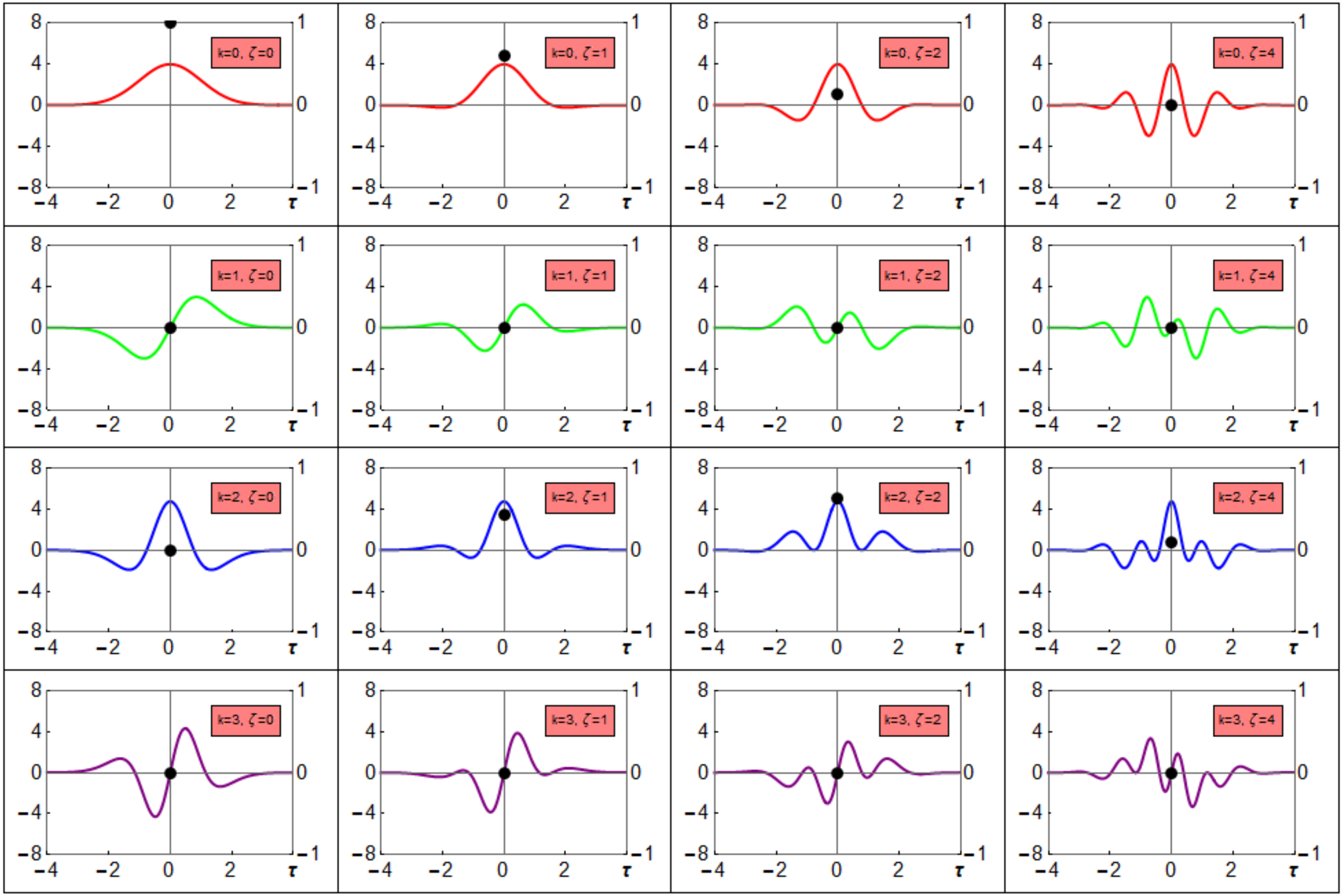


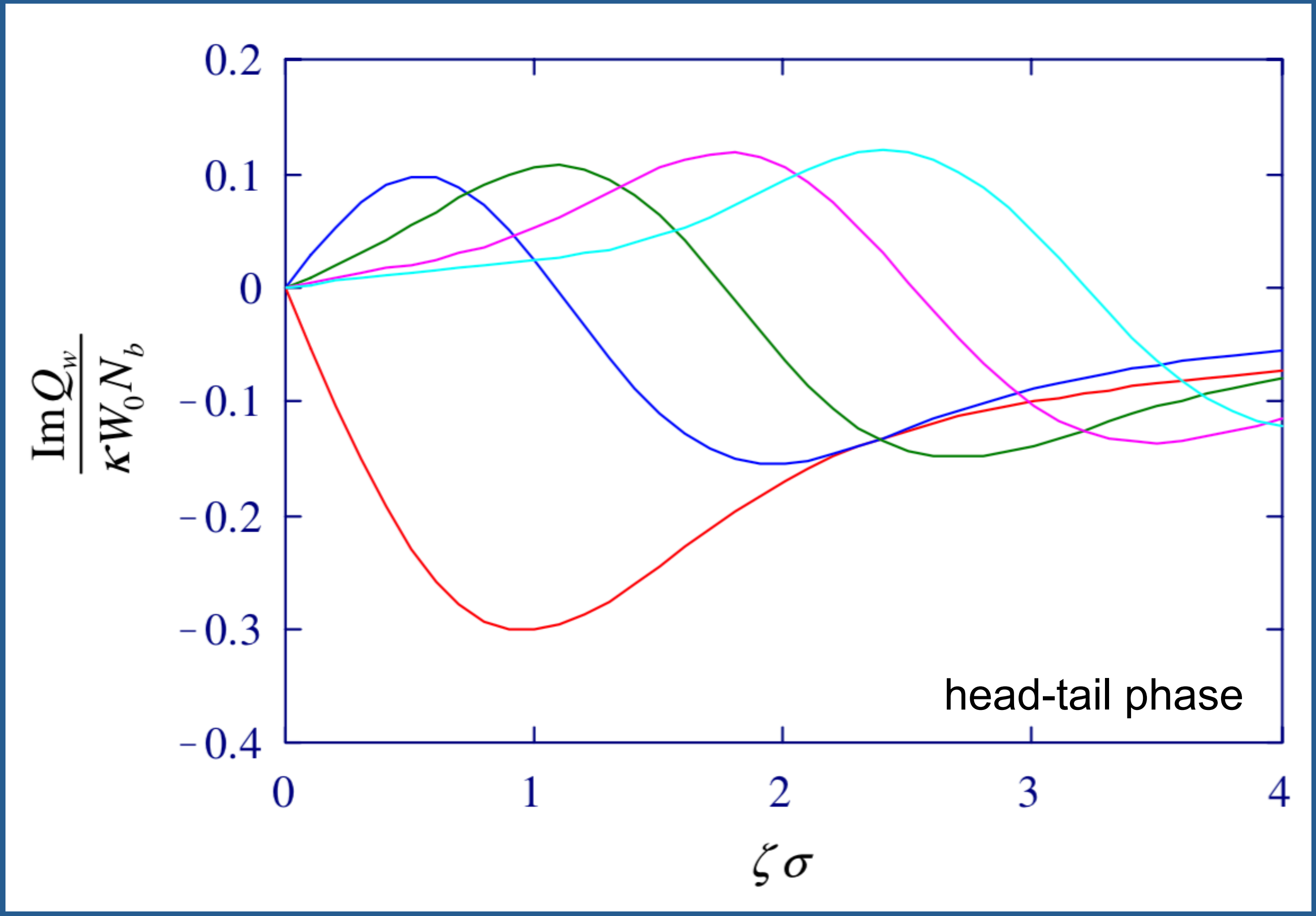
TABLE I. Eigenvalues and asymptotic values for the first ten modes of the Gaussian bunch.

k	0	1	2	3	4	5	6	7	8	9
ν	0	1.4	4.4	8.9	15	23	32	43	56	70
$\lvert y_\infty \rvert$	1	1.6	2.0	2.3	2.5	2.7	2.9	3.0	3.2	3.3

No-Wake Eigen-system



Weak Head-Tail Growth Rates



Rates are qualitatively similar to no-space-charge case

Nonlinear LD

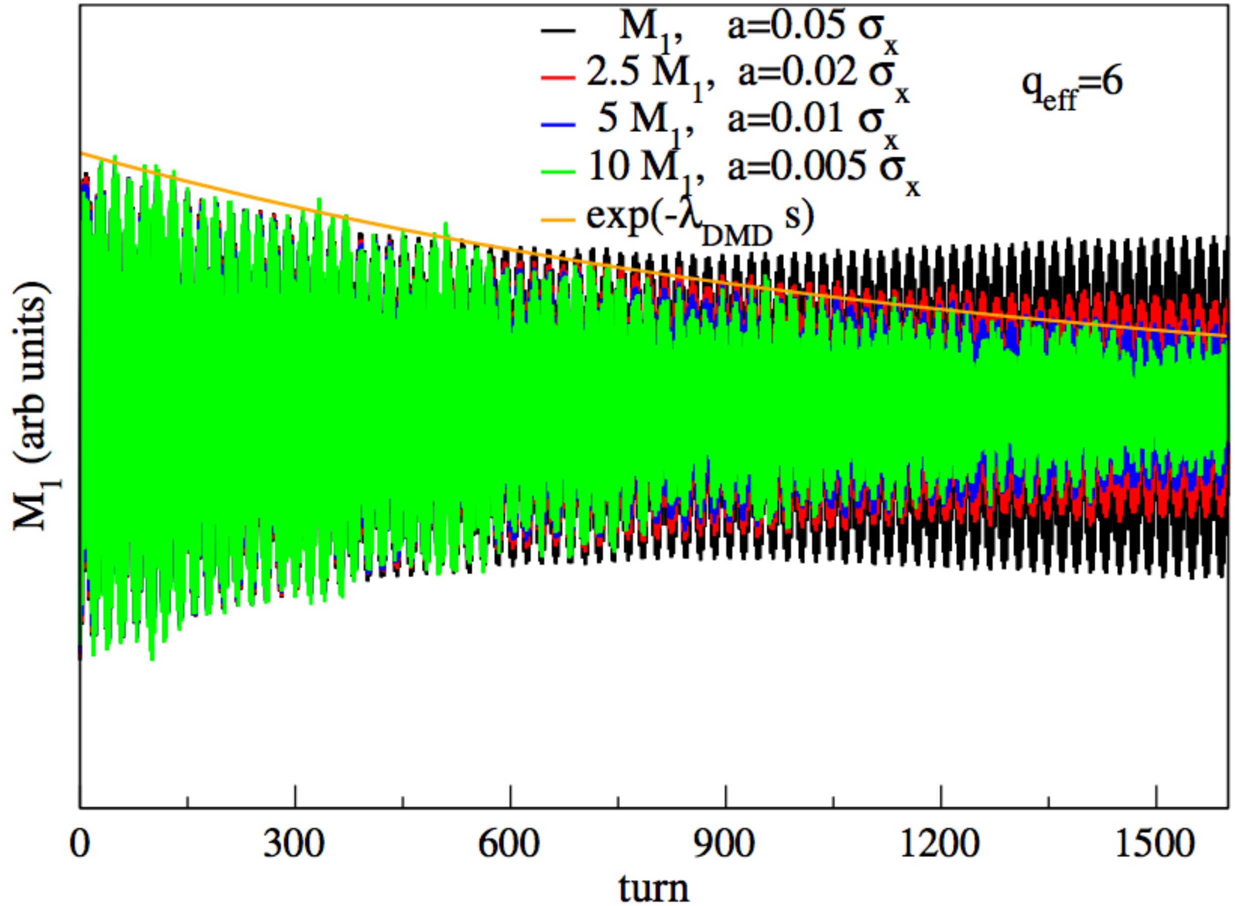


FIG. 10. 3D-G bunch. M_1, Eq. (36), versus turn number at $q_{\rm eff} = 6$ for different excitation amplitudes. M_1 is multiplied by a factor inverse proportional to the excitation amplitude. The linear regime requires an excitation amplitude smaller than $0.01\sigma_x$. The exponential decay in the linear regime is consistent with the one provided by the DMD analysis. In the nonlinear regime the long-time behavior is very sensitive to the excitation amplitude.