Head-Tail Instability and Landau Damping in Bunches with Space Charge

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Coherent Eigenmodes

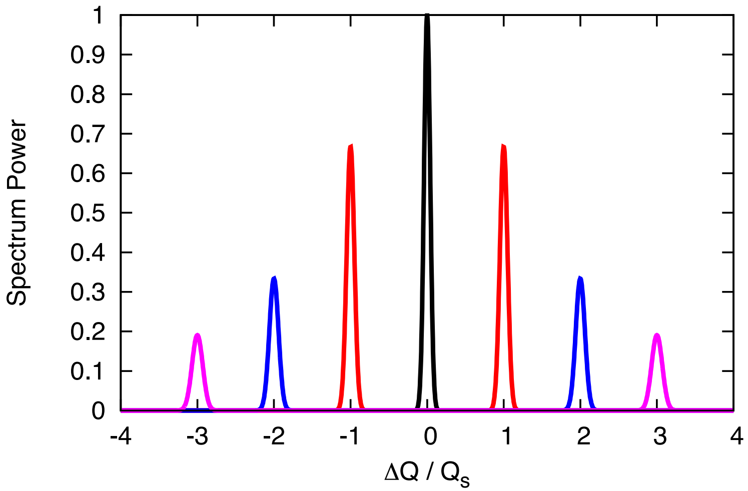
The real part of the mode frequency is not only the spectrum, it defines:

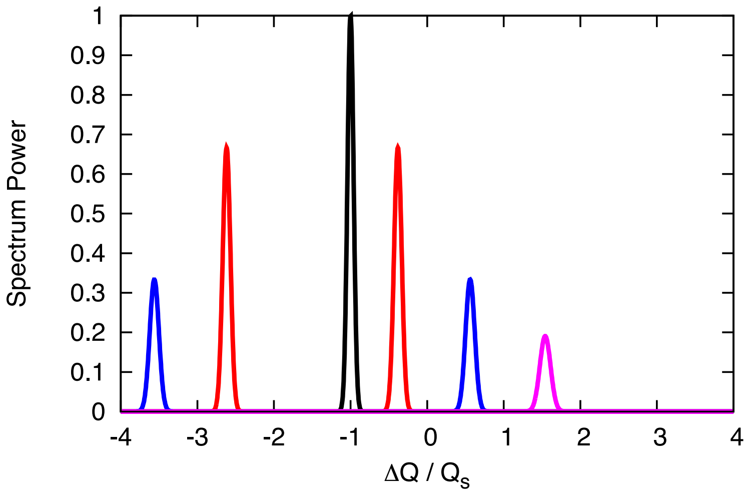
- the instability drive (↔ impedances)
- the Landau damping (↔ incoherent tune spreads)

$$\Delta\Omega = \Delta\Omega_{\mathrm{Re}} + i\gamma_{\mathrm{drive}} + i\gamma_{\mathrm{damping}}$$

Coherent Bunch Eigenmodes

k=−3 k=−2 k=−1 k=0 k=1 k=2 k=3





$$\Delta Q = \frac{\Delta f}{f_0} = \frac{f - (p + Q_0) f_0}{f_0}$$

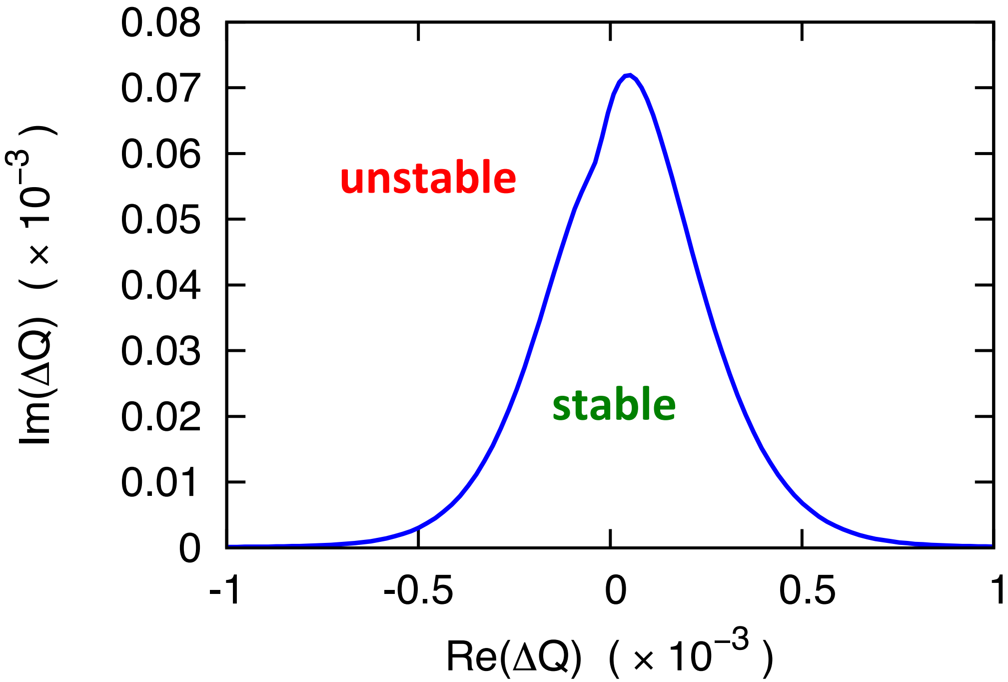
The coherent lines:

- Shifted by impedances, and by space-charge
- Interact (driven) with impedances Z(f)
- Interact with individual (incoherent) particle oscillations

Landau Damping: Dispersion Relation

complex coherent tune shift ΔQ for the beam without damping

The solution: collective mode frequency Ω for the given impedance and beam



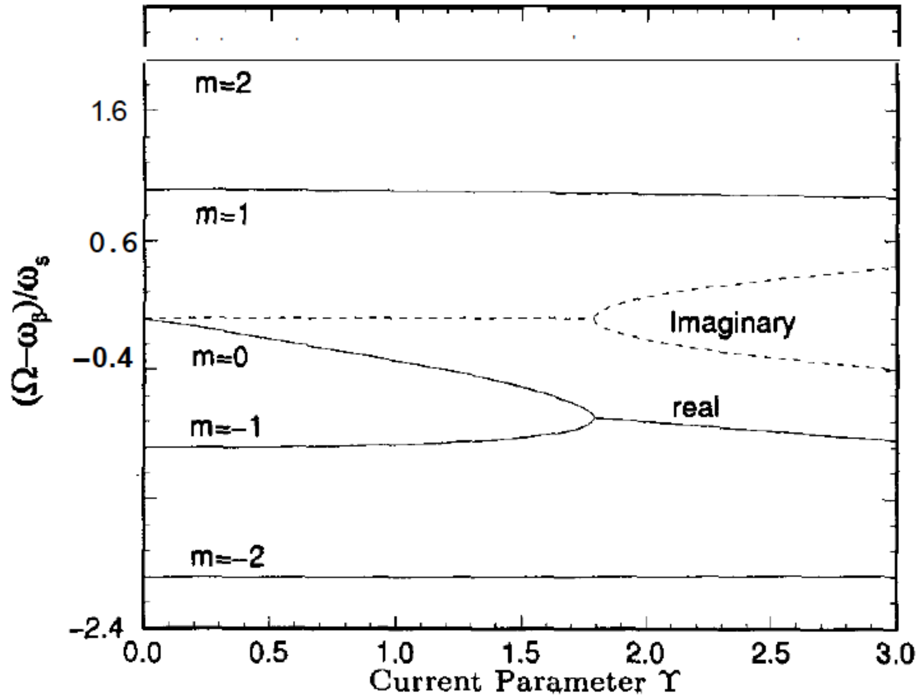
Approach so far:
The coasting beam dispersion relation, should be adequate for the k=0 mode

Accurate predictions of the coherent tune shifts are essential for the instability thresholds, intensity limits, feedback design

Bunch Eigenfrequencies

The coherent tune shift ΔQ can be small
$Q_0 \gg Q_s$
and not very important for Z(f)

but the modes can couple
and produce the Transverse Mode Coupling Instability (TMCI)



K.Y.Ng, Physics of Intensity Dependent Beam Instabilities, 2006

Accurate predictions of the coherent tune shifts are essential for the instability thresholds, intensity limits, feedback design

Airbag Bunch Model

A bunch model with the barrier potential, rigid slices:
Analytical solution for arbitrary space-charge
M.Blaskiewicz, PRSTAB **1**, 044201 (1998)

$$\Delta Q_k = -\frac{\Delta Q_{\mathrm{sc}}}{2} \pm \sqrt{\frac{\Delta Q_{\mathrm{sc}}^2}{4} + k^2 Q_s^2}$$

Extended with a coherent force (the same derivation) in
O.Boine-Frankenheim, V.Kornilov, PRSTAB **12**, 114201 (2009)

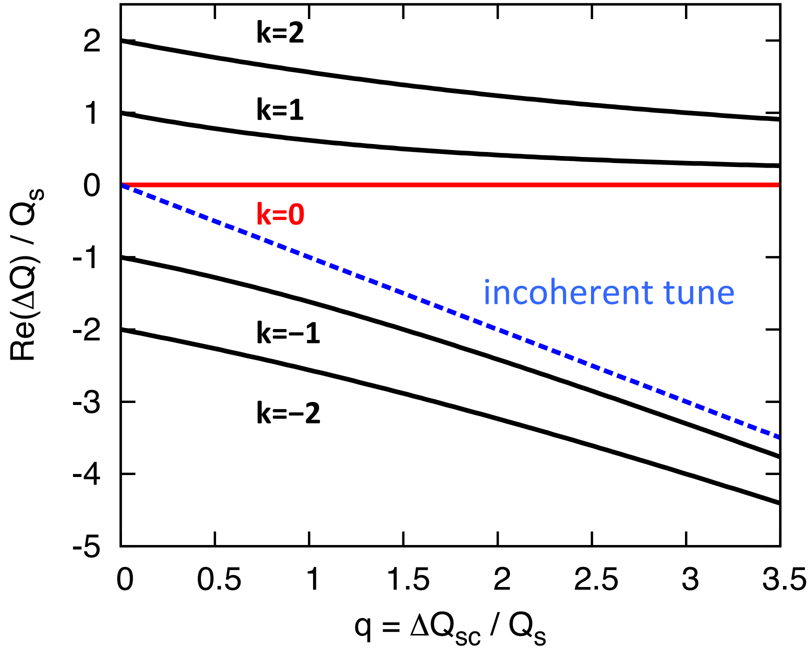
$$\Delta Q_k = -\frac{\Delta Q_{\mathrm{sc}} + \Delta Q_{\mathrm{coh}}}{2} \pm \sqrt{\frac{(\Delta Q_{\mathrm{sc}} - \Delta Q_{\mathrm{coh}})^2}{4} + k^2 Q_s^2}$$

However, scepticism has been expressed recently.

Airbag Bunch Model

Effect of Space-Charge

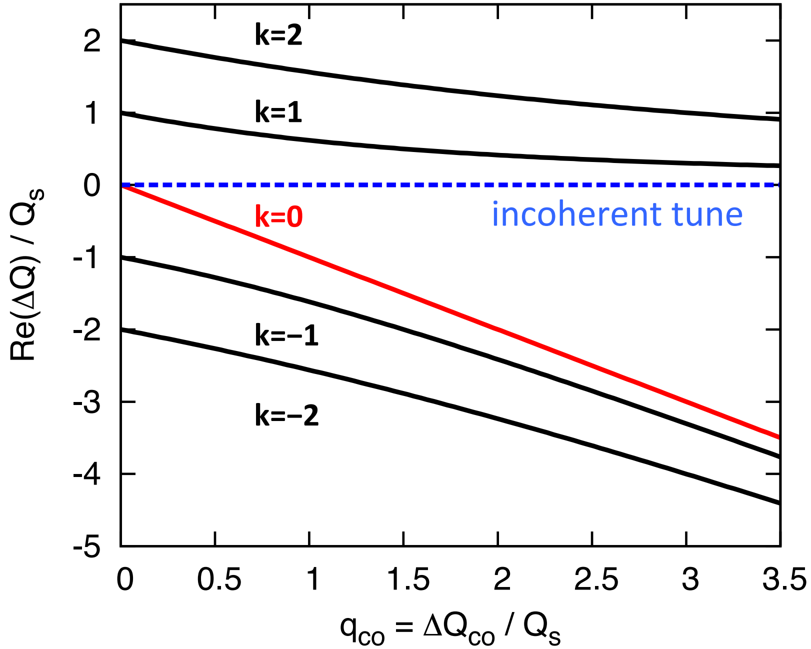
$$q = \frac{\Delta Q_{\mathrm{sc}}}{Q_s}$$



k=0 mode not affected
incoh. tune shifted

Effect of an impedance

$$q_{\mathrm{co}} = \frac{\Delta Q_{\mathrm{coh}}}{Q_s}$$



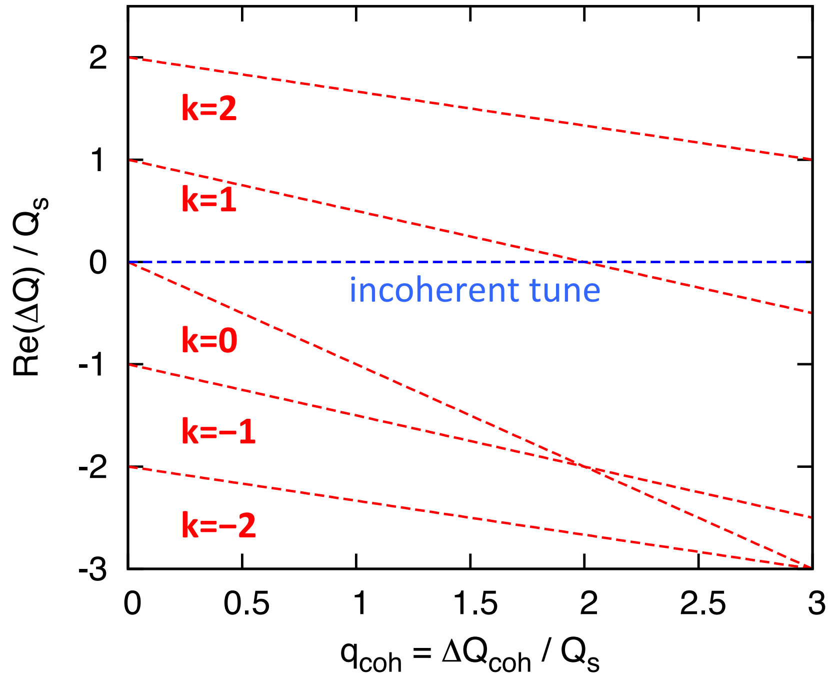
k=0 mode shifted
inc. tune not affected

Shifts of Head-Tail Modes

The standard model for decades:
The theory of F. Sacherer 1974

$$\Delta Q_k = -\frac{\Upsilon}{1+k}\frac{\sum iZ_\perp(\omega_p)h_k(\omega_p-\omega_\xi)}{\sum h_k(\omega_p-\omega_\xi)}$$

$$\omega_p = (p+Q_0)\omega_0 + k\omega_p$$



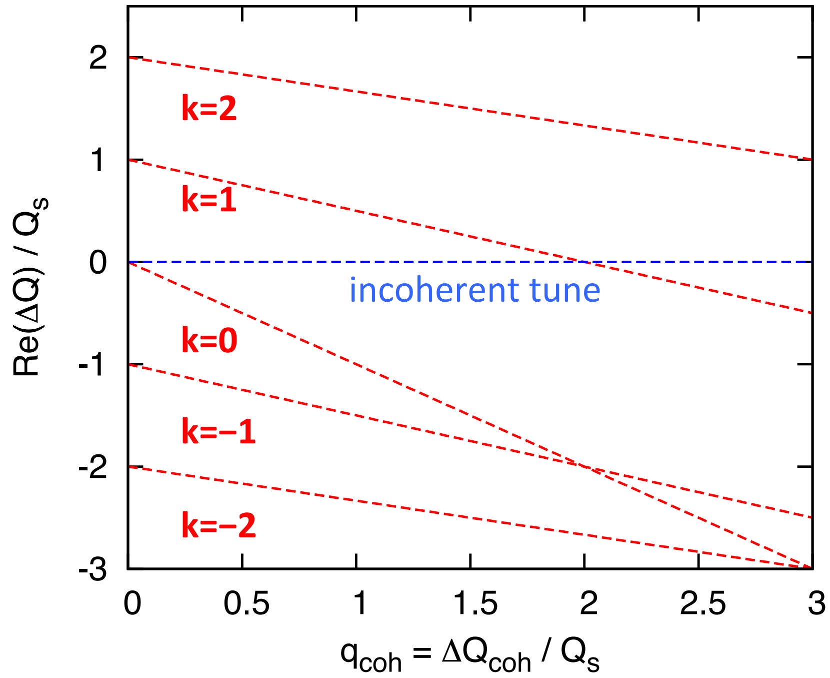
Space-Charge effect not included

Shifts of Head-Tail Modes

The standard model for decades:
The theory of F. Sacherer 1974

$$\Delta Q_k = -\frac{\Upsilon}{1+k}\underbrace{\frac{\sum iZ_\perp(\omega_p)h_k(\omega_p-\omega_\xi)}{\sum h_k(\omega_p-\omega_\xi)}}_{\Delta Q_{\rm coh}}$$

$$\omega_p = (p+Q_0)\omega_0 + k\omega_p$$



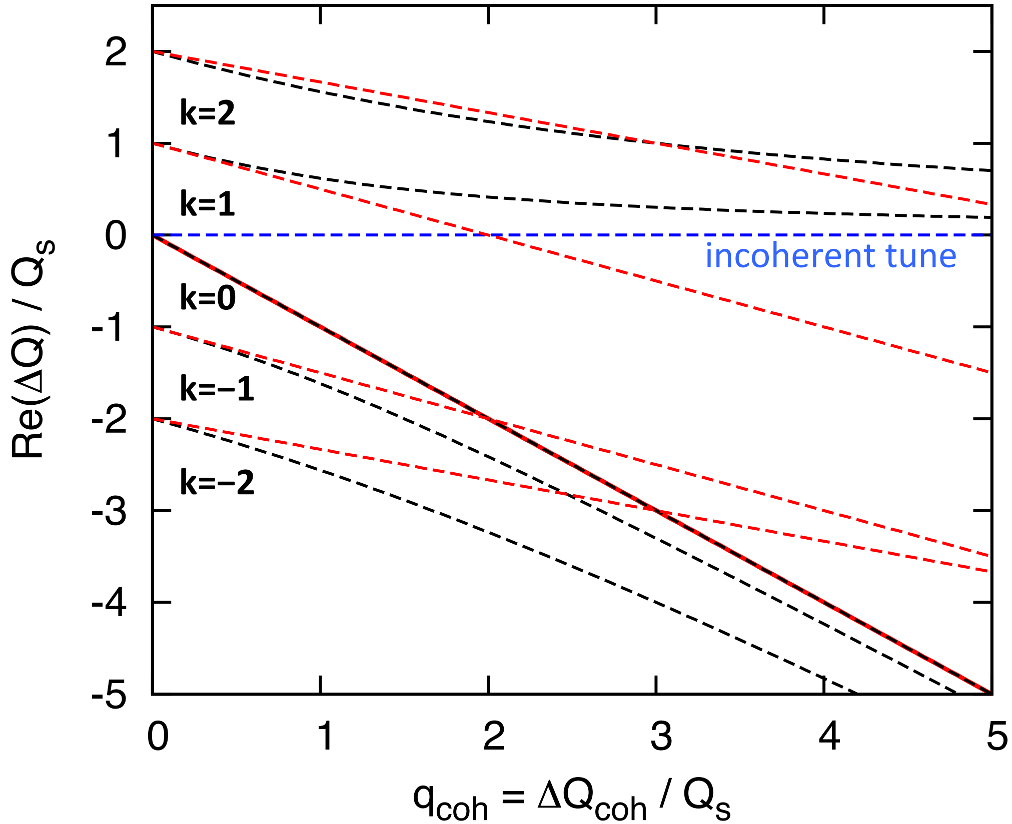
For a broadband impedance:

$$\Delta Q_k = -\frac{\Delta Q_{\rm coh}}{1+k}$$

Space-Charge effect not included

Shifts of Head-Tail Modes

Sacherer Model (red lines)
vs
Airbag Model (black lines)



strong differences

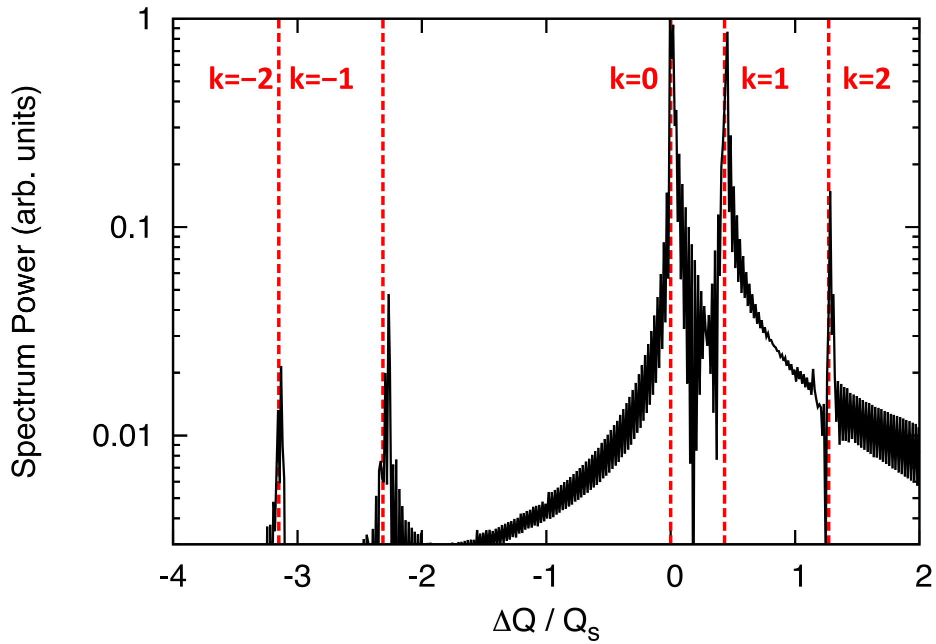
Particle Tracking Simulations

The PIC code PATRIC

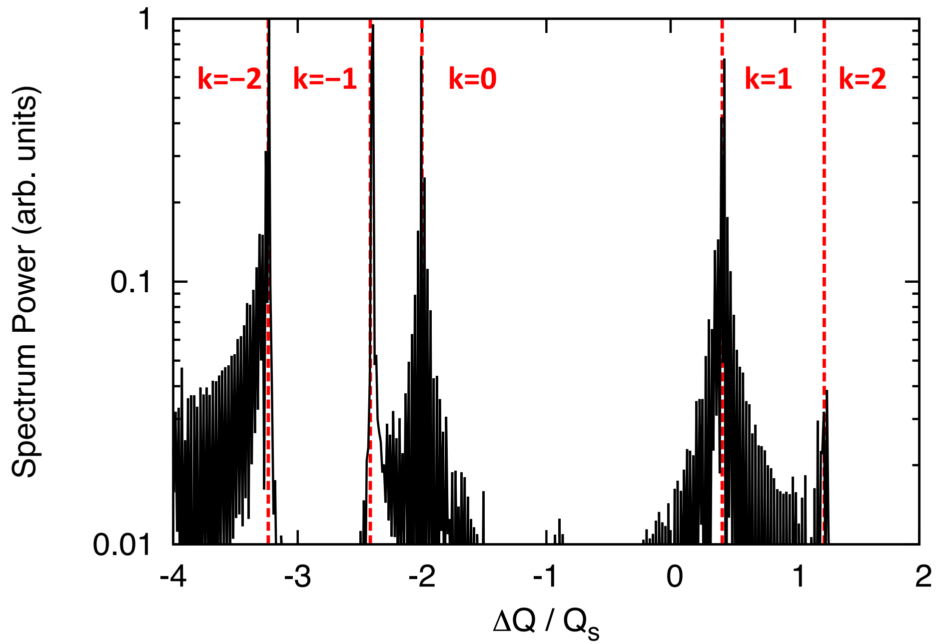
- 2.5D sliced bunches
- Self-consistent space-charge, frozen space-charge
- Impedances, Wakes
- Snapshot domain (space), fixed-location domain (time)
- Tune shifts, spectra, instabilities verified with analytical theories:
  V. Kornilov and O. Boine-Frankenheim, Proc. of ICAP2009, San Francisco (2009)
  O.Boine-Frankenheim, V.Kornilov, Proc. of ICAP2006 (2006)
- Verified vs. HEADTAIL (CERN)
- Landau damping simulations, head-tail modes with space-charge:
  V.Kornilov, O.Boine-Frankenheim, PRSTAB **13**, 114201 (2010)

Shifts of Head-Tail Modes

Effect of Space-Charge
$q=\Delta Q/Q_s=2$



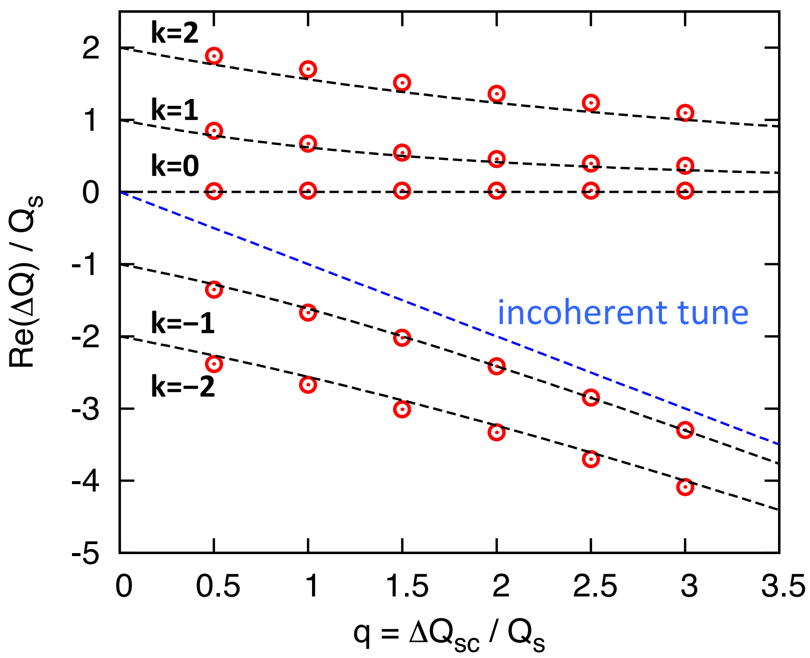
Effect of an impedance
$q_{co}=\Delta Q_{co}/Q_s=2$



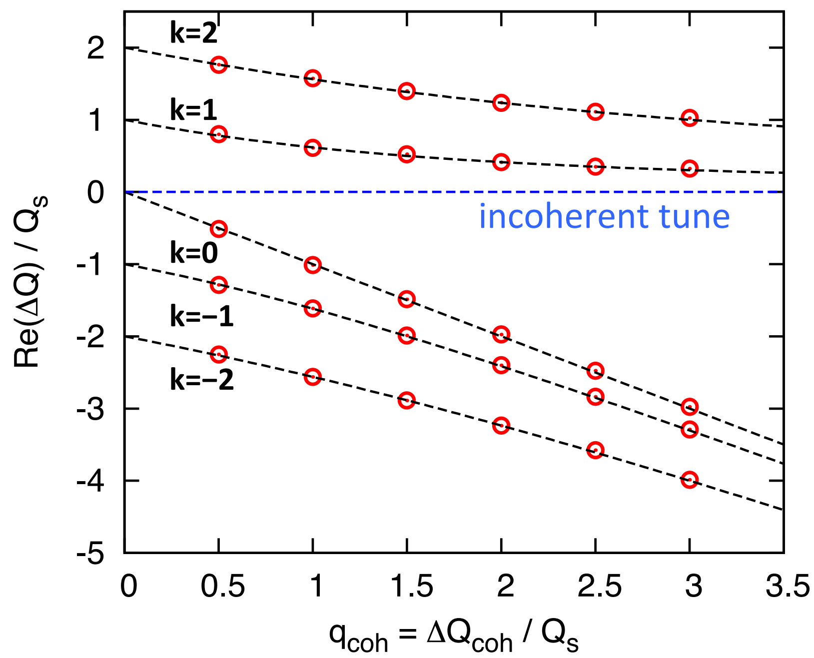
Agreement between
the particle tracking simulations (black line) and
the airbag theory (red lines)

Shifts of Head-Tail Modes

Effect of Space-Charge



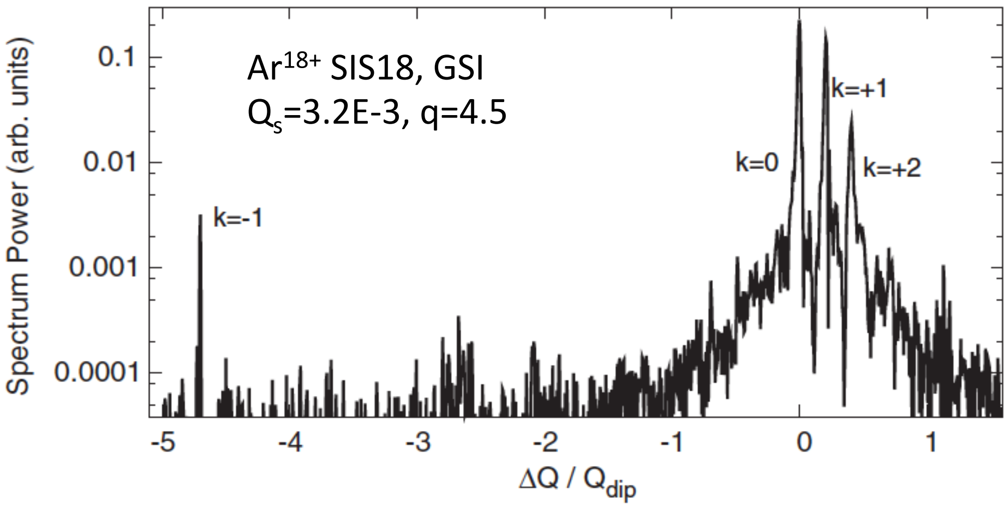
Effect of an impedance

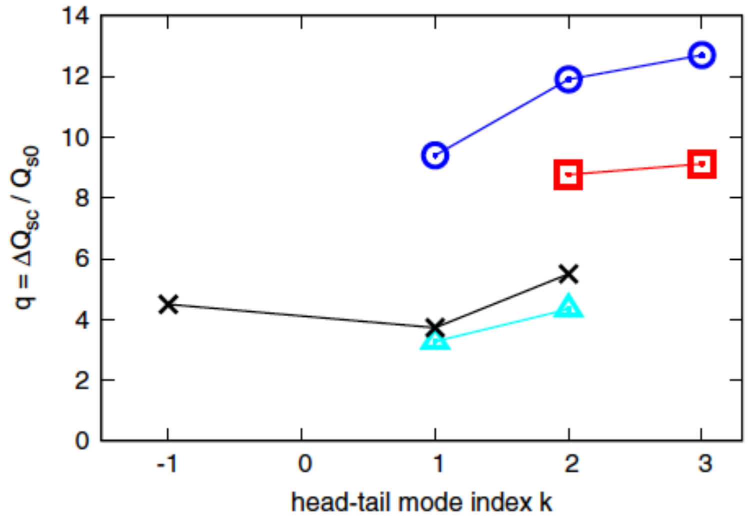


Agreement between
the particle tracking simulations (red circles) and
the airbag theory (black lines)

Shifts of Head-Tail Modes

Confirmations by the experiment



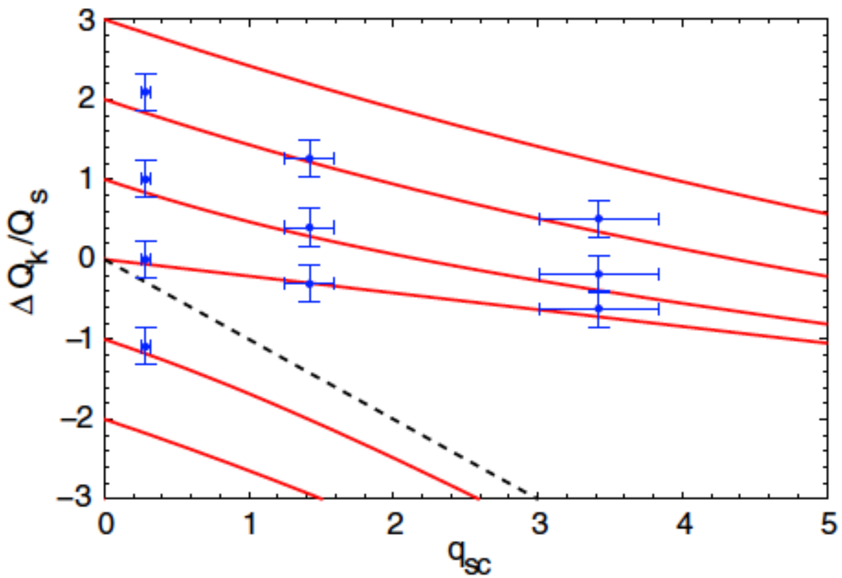


V.Kornilov, O.Boine-Frankenheim, PRSTAB 15, 114201 (2012)

Agreement between the experiment and the airbag theory

Shifts of Head-Tail Modes

Confirmations by the experiment

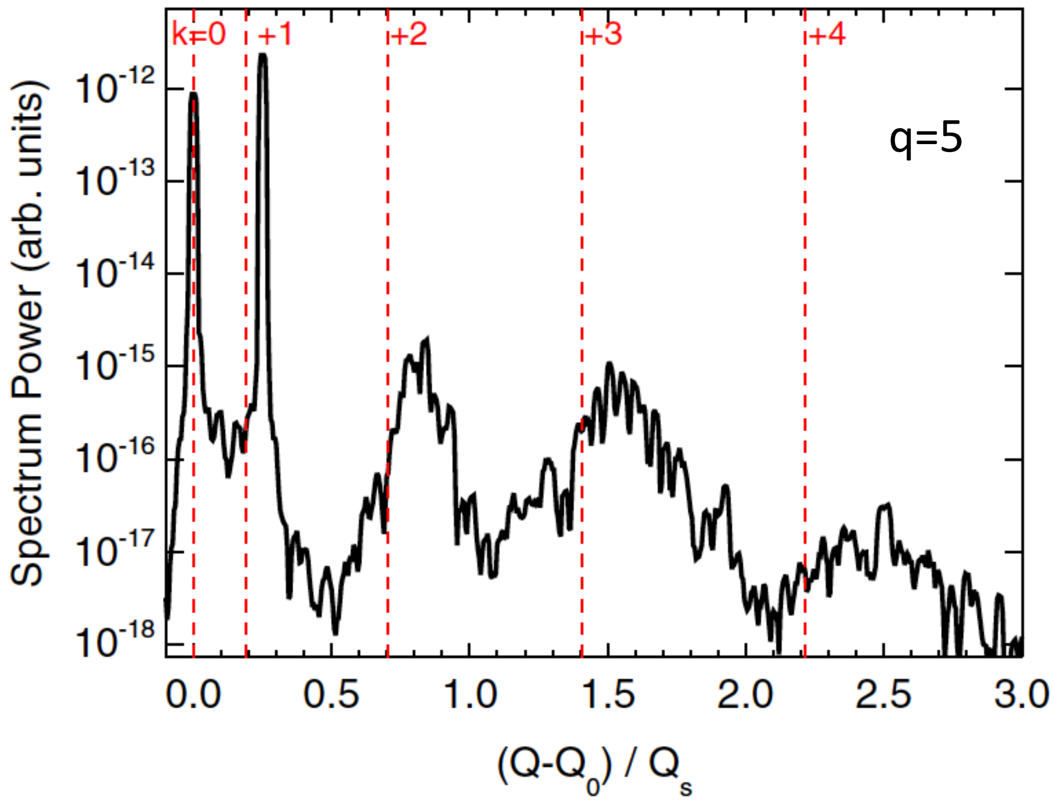


R.Singh,et.al, PRSTAB 16, 034201 (2013)

Agreement between the experiment and the airbag theory

Shifts of Head-Tail Modes

Gaussian (longitudinal and transverse) bunch:
Tune shifts are close to the airbag predictions

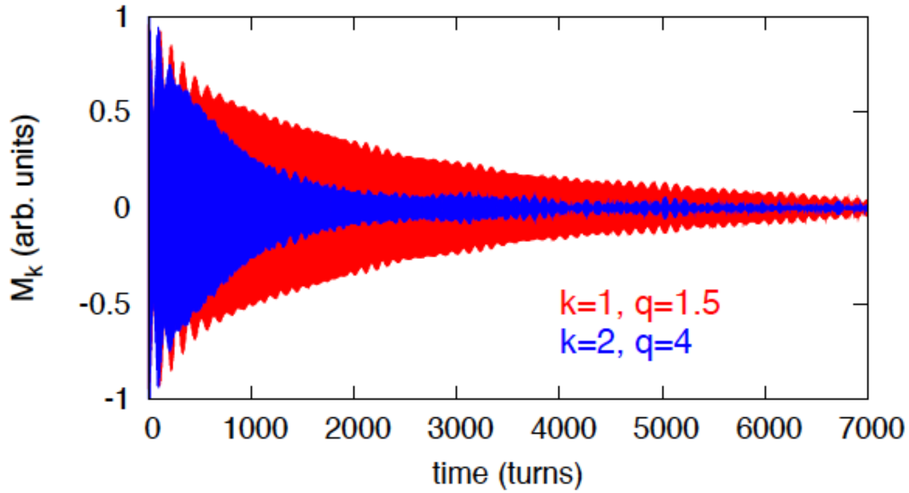


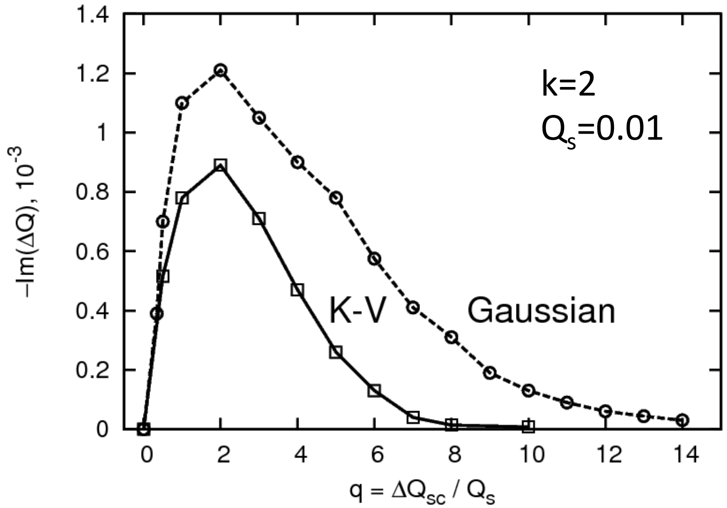
particle tracking simulations (the black line) and the airbag theory (the red lines)

Landau Damping in Bunches

Landau damping in bunches
exclusively due to the effect of space charge

Burov, PRSTAB 2009, Balbekov, PRSTAB 2009,
V.Kornilov, O.Boine-Frankenheim, PRSTAB 2010



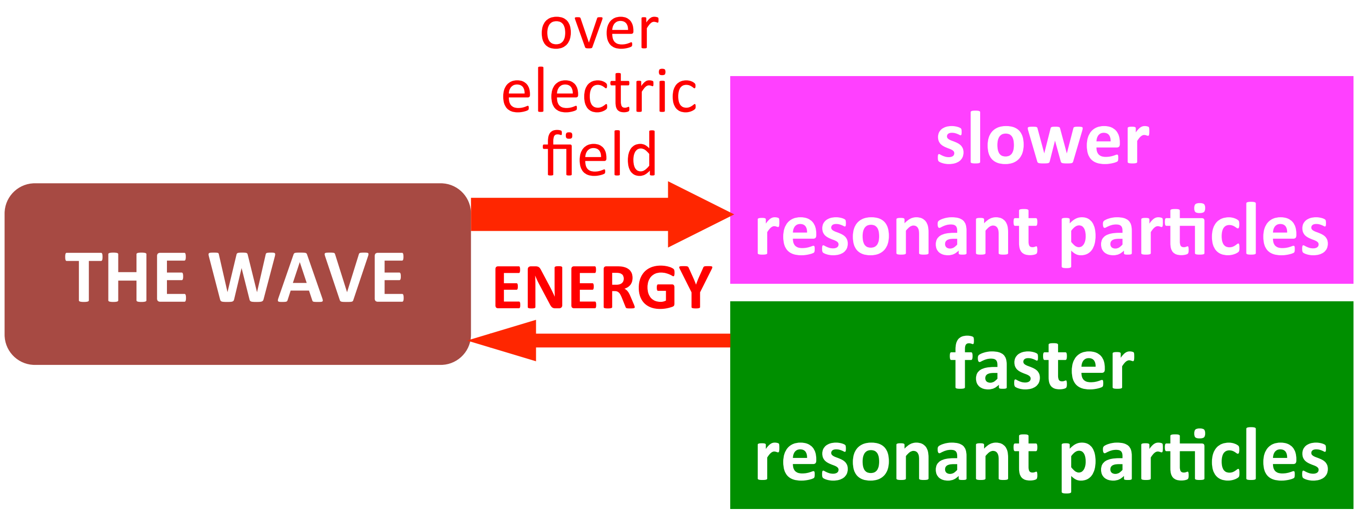


Is this damping important?
Yes! Typical instability observed $\Delta Q/Q_s$: 0.1…0.2 (PSB, ISIS)

PSB: CERN-ACC-NOTE-2014-0025 (2013)
ISIS: HB2014

Landau Damping due to Space-Charge

Basically, very similar to Landau damping in plasma:



$$\partial f / \partial J_x \Big|_{\omega=\Omega_{\mathrm{coh}}}$$

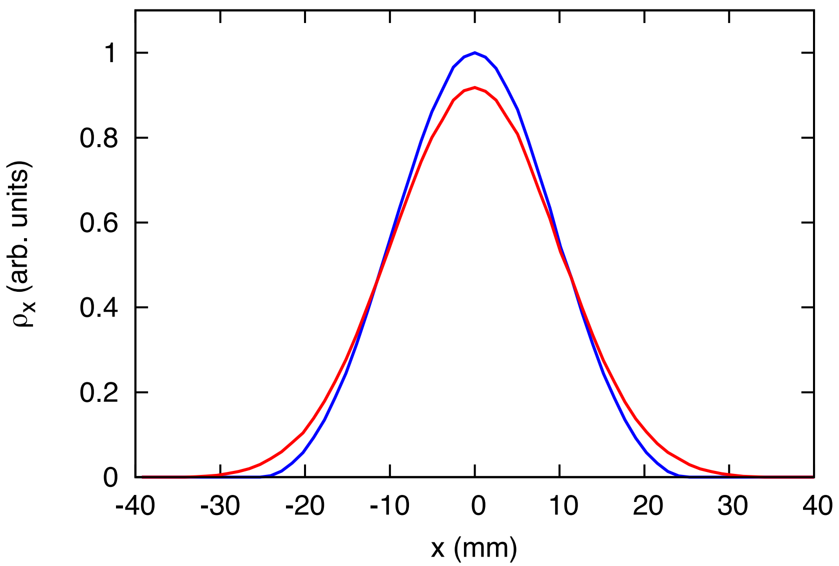
Main ingredients of Landau damping:

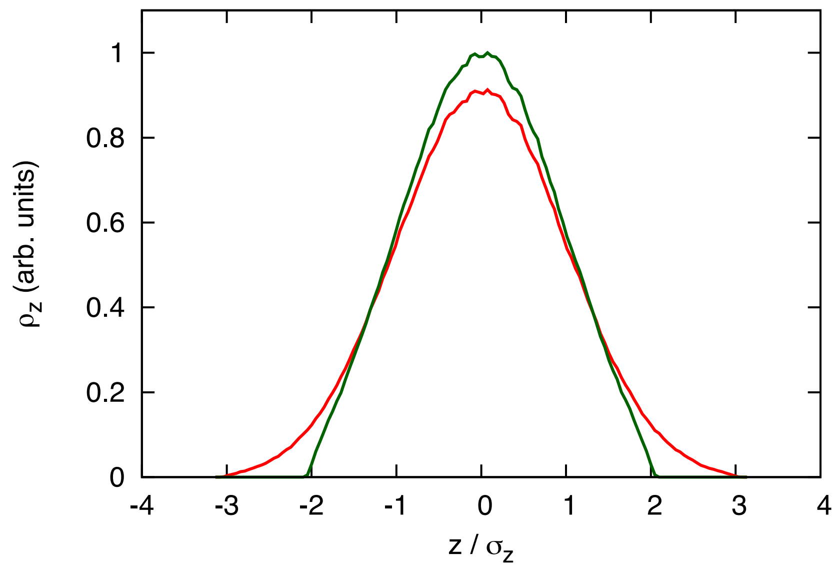
✓ wave−particle collisionless interaction: ***E***-field of Space-Charge

✓ energy transfer: the wave ↔ the (few) resonant particles

Landau Damping

The resonant particles are in the distribution tails
Thus:
Landau damping depends on the transverse/longit distribution



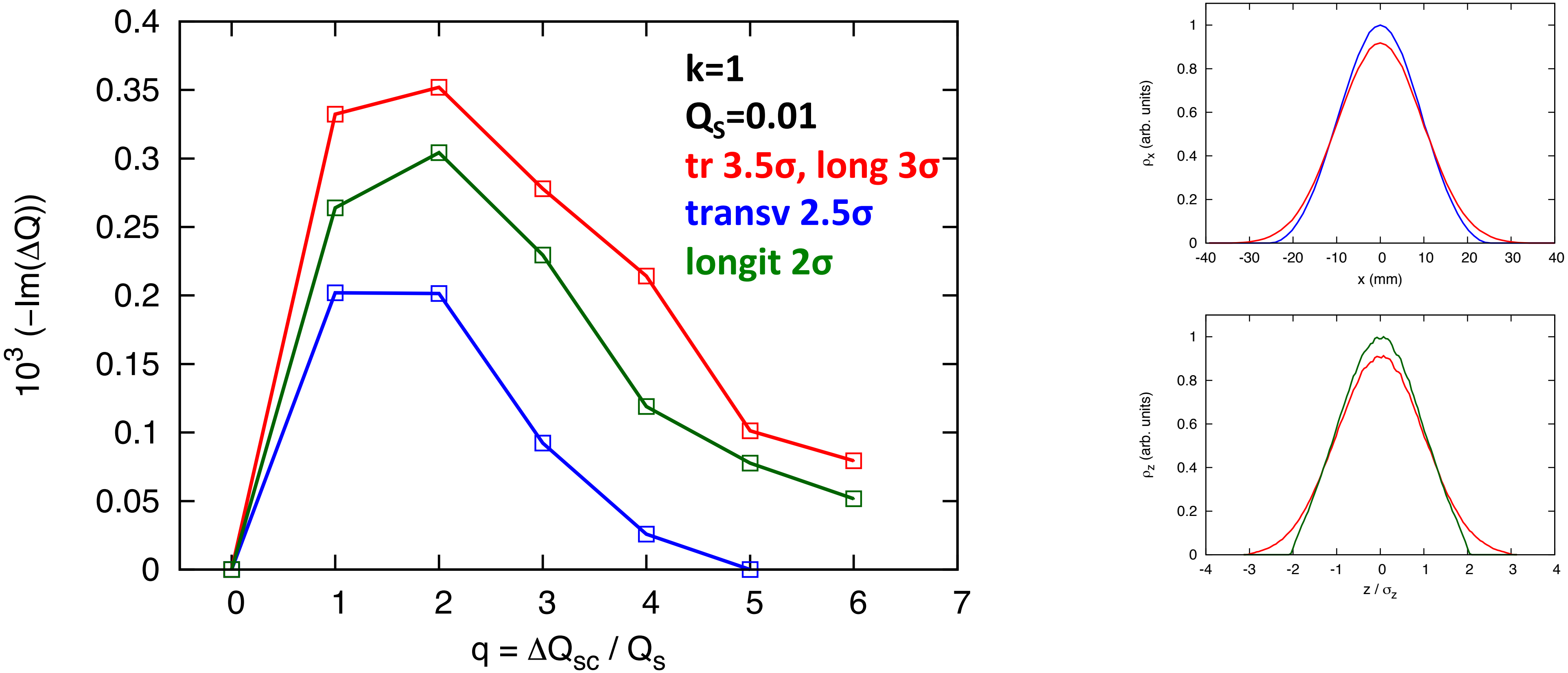


Transverse
Red: truncate at 3.5σ
Blue: truncate at 2.5σ

Longitudinal
Red: truncate at 3σ
Green: truncate at 2σ

Landau Damping

The resonant particles are in the distribution tails.
Landau damping depends on the transverse/longit distribution.



Damping rate from the particle tracking simulations

Model for Landau Damping

Resonant particles for the effective Landau damping

$$\Delta Q_\eta = -\eta \Delta Q_{\rm sc}$$

Modulated coherent frequency

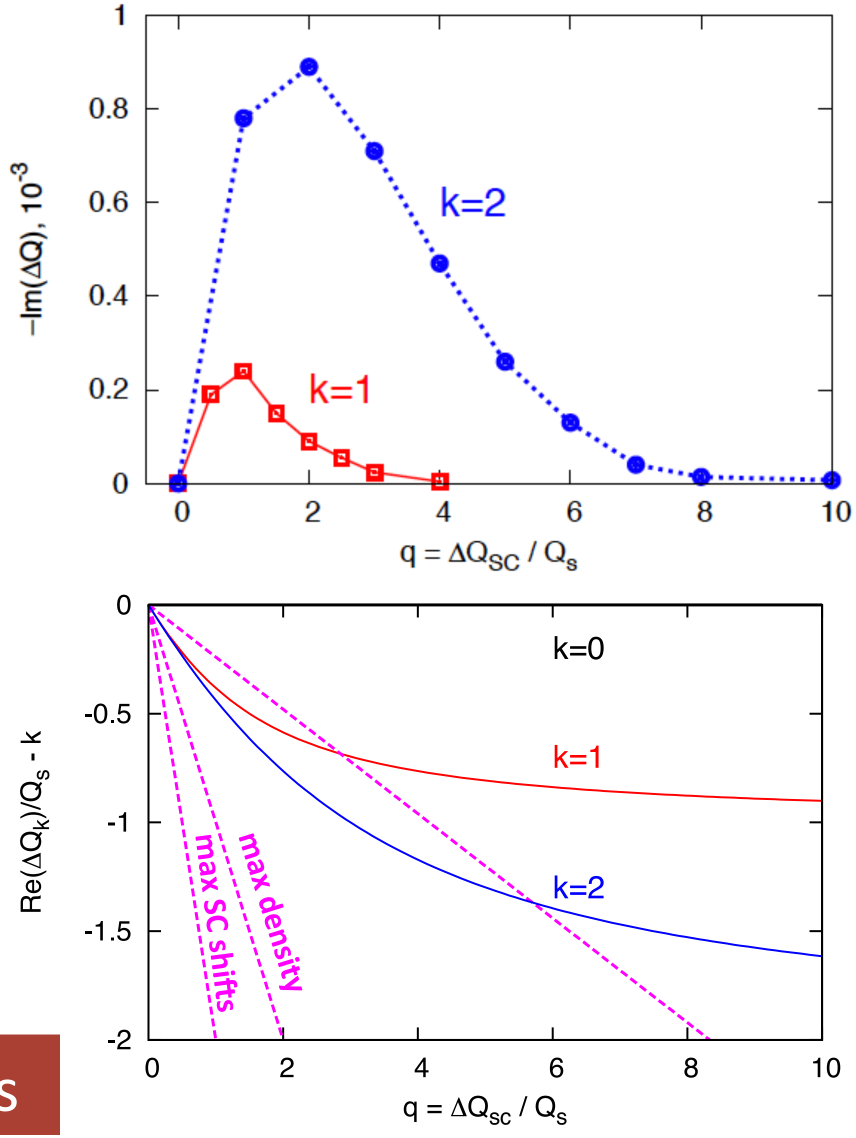
$$\Delta Q_k - k Q_s$$

Resulting damping range

$$q = k \frac{1-2\eta}{\eta(1-\eta)}$$

Here η=0.24, q_{max}=2.8 (k=1) and q_{max}=5.7 (k=2)

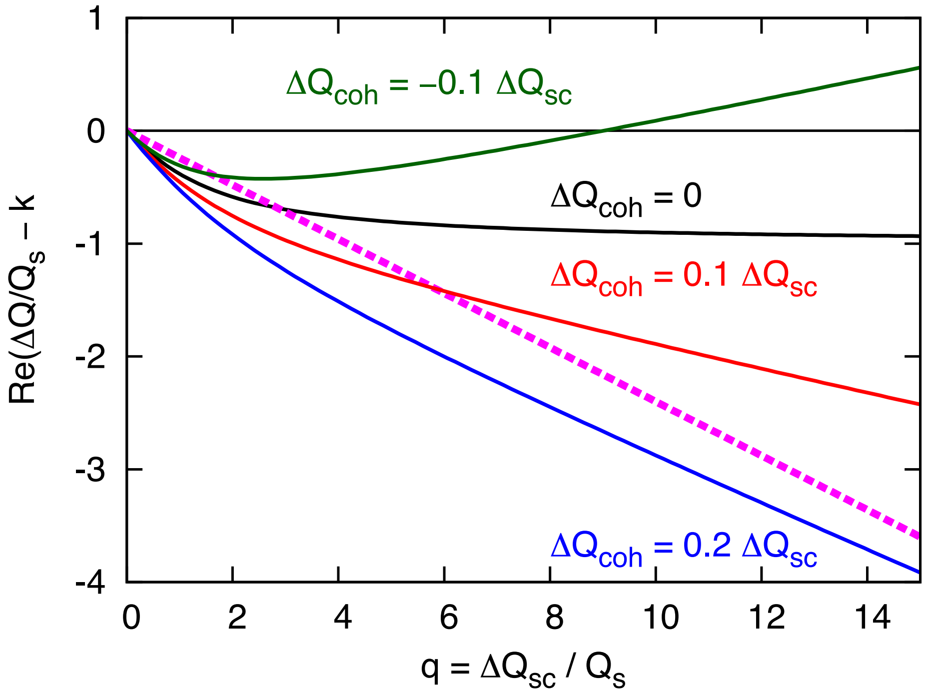
Good agreement with the simulations



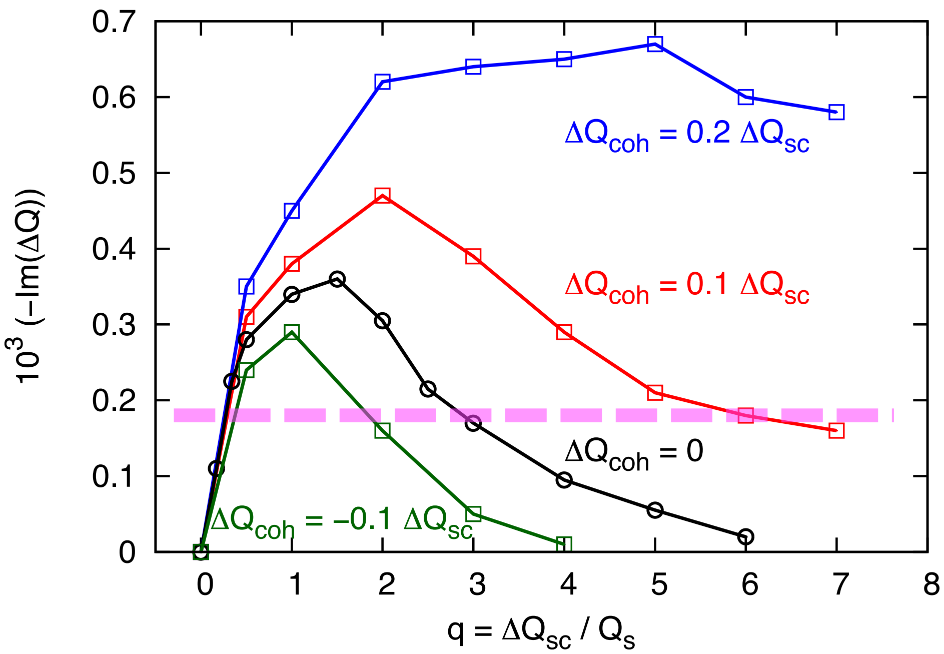
Model for Landau Damping

Effect of an impedance

Landau damping model



PIC Simulations

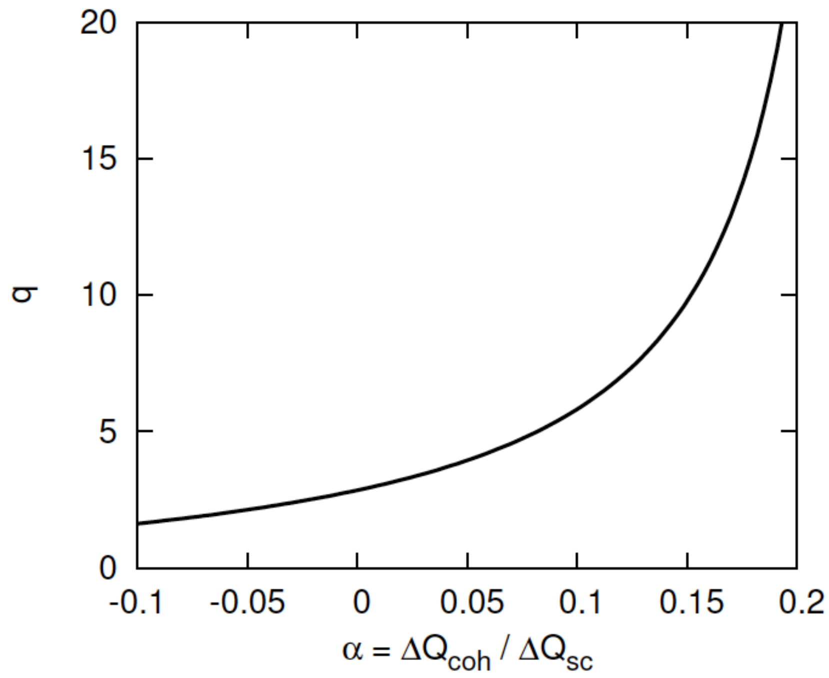


Good agreement with the simulations:
coherent shifts enhance (weaken) Landau damping

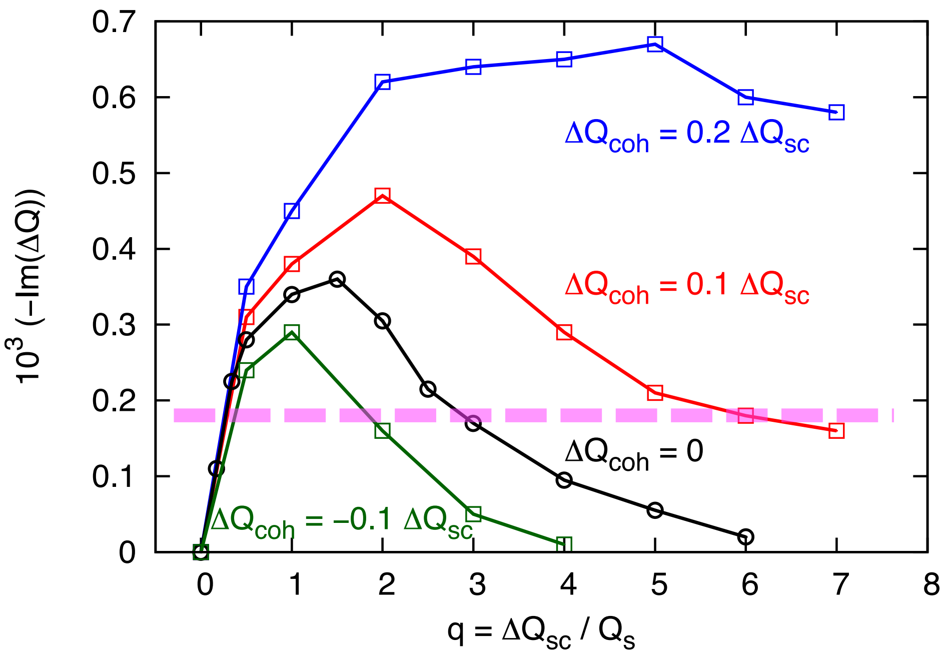
Model for Landau Damping

$$q = k\frac{\alpha + 1 - 2\eta}{(\eta - \alpha)(1 - \eta)}$$

Landau damping model



PIC Simulations

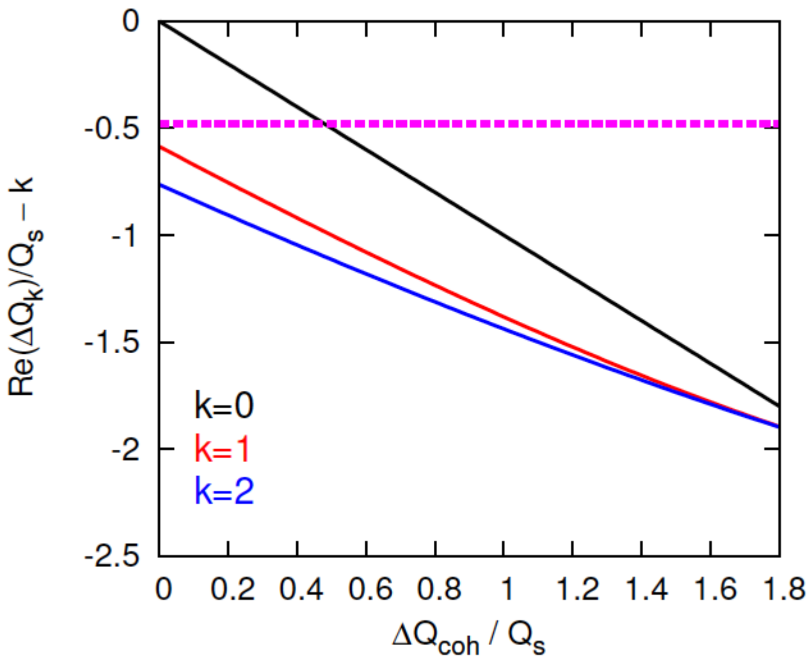


Good agreement with the simulations:
effect of coherent shifts on the effective damping range

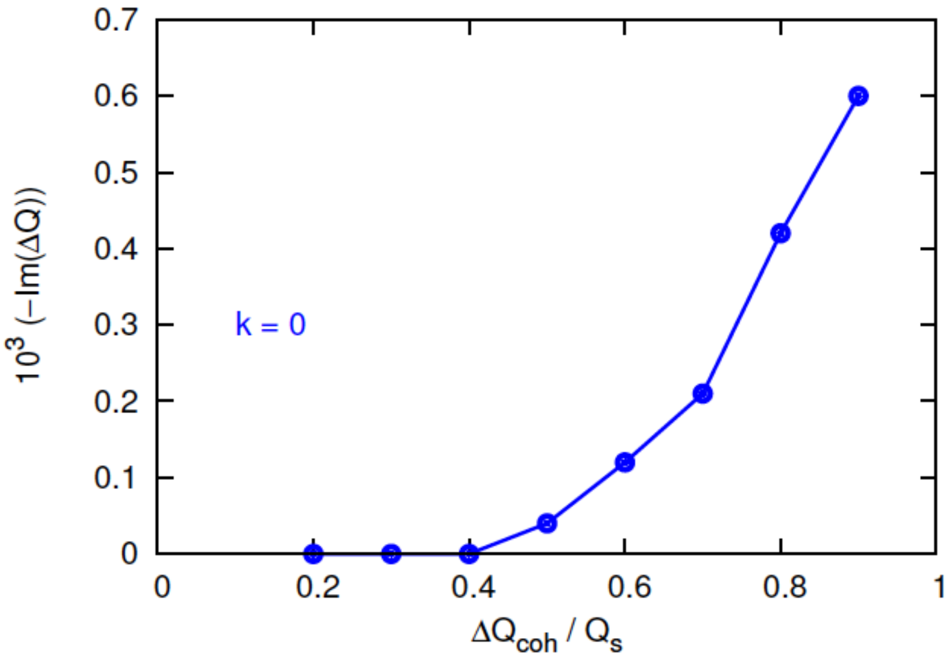
Model for Landau Damping

Coherent shifts due to impedance

Landau damping model



PIC Simulations



Good agreement with the simulations:
Landau damping of the k=0 mode due to space-charge

Conclusions & Outlook

- Landau damping is the essential part of the beam stability
- We are now able to predict the instability thresholds: accurate $\mathrm{Re}(\Delta Q_{coh})$ and incoherent (inter. & exter.) spectrum needed
- The airbag theory for head-tail shifts due to space-charge, due to coherent effect, and the combinations, is verified by simulations and by the experiment
- The model of the effective Landau damping with the modulated coherent frequency gives correct predictions, and adequate physical understanding

Conclusions & Outlook

The dispersion relation

$$\int \frac{\Delta Q_{\rm coh} - \Delta Q_{\rm sc}}{\Delta Q_{\rm ex} + \Delta Q_{\rm sc} - \Omega/\omega_0} J_x \frac{\partial f}{\partial J_x} \mathrm{d}J_x \mathrm{d}J_y \mathrm{d}p = 1$$

$f(J_x, J_y, p)$

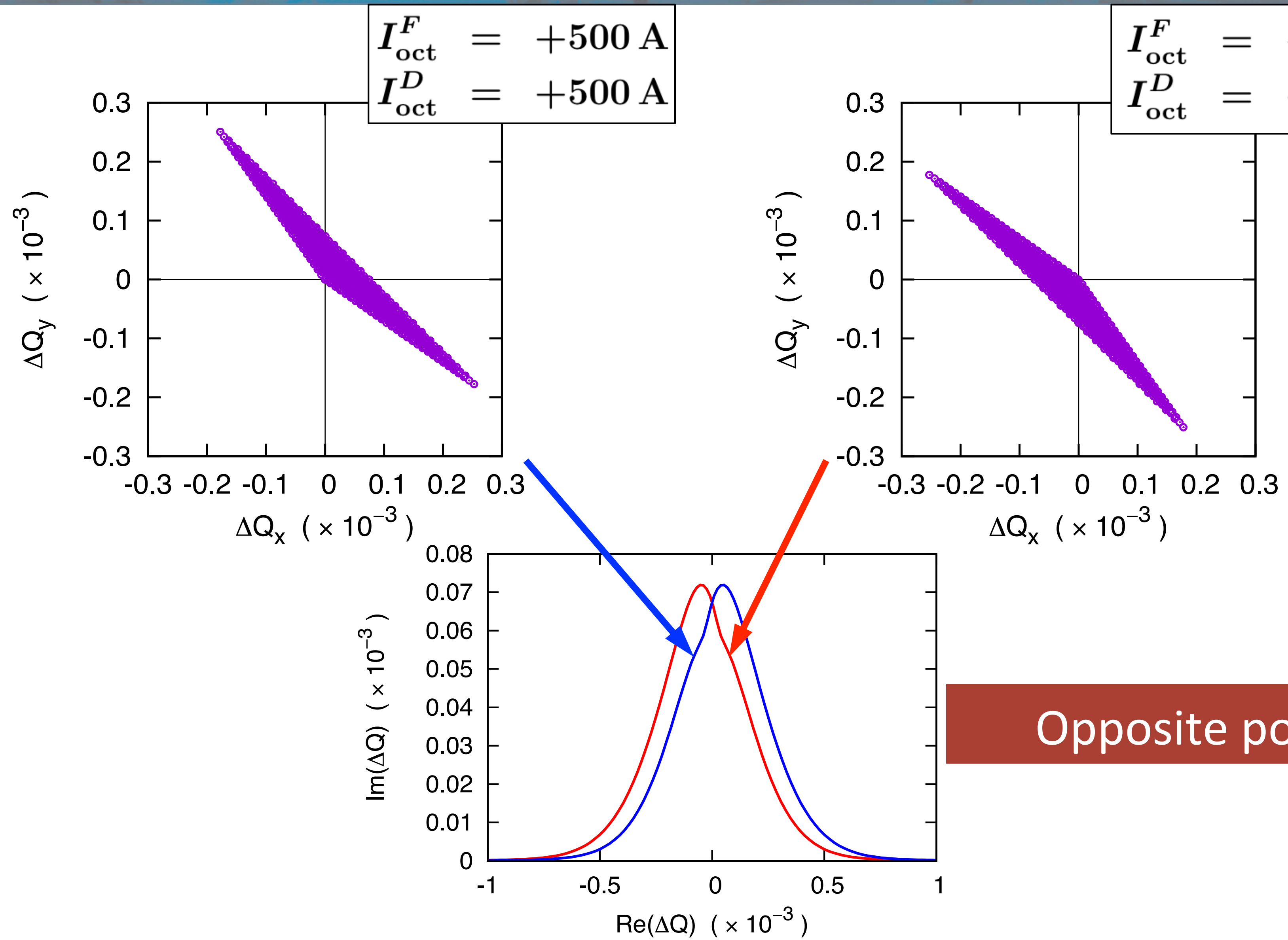
ΔQ_{coh} : no-damping coherent tune shift imposed

$\Delta Q_{ex}(J_x, J_y, p)$: external (lattice) incoherent tune shift

$\Delta Q_{sc}(J_x, J_y)$: space-charge tune shift

L.Laslett, V.Neil, A.Sessler, 1965
D.Möhl, H.Schönauer, 1974

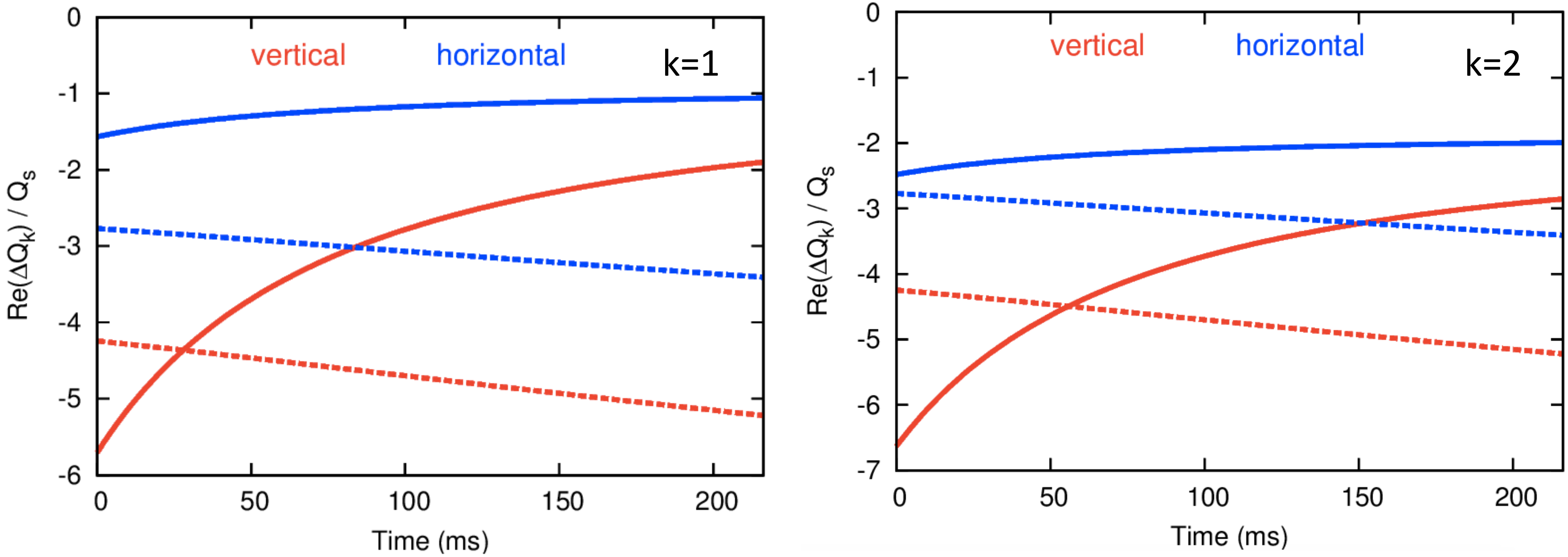
The resulting damping is a complicated 2D convolution of the distribution $\{df(J_x,J_y)/dJ_x\}$ and tune shifts $\Delta Q_{sc}(J_x,J_y)$, $\Delta Q_{ext}(J_x,J_y)$



Opposite polarity

DAMPING IN SIS100 BUNCHES

$$\Delta Q_k(t) = -\frac{\Delta Q_{\rm sc}(t) + \Delta Q_{\rm coh}(t)}{2} \pm \sqrt{\frac{[\Delta Q_{\rm sc}(t) - \Delta Q_{\rm coh}(t)]^2}{4} + k^2 Q_s^2(t)}$$



- predictions from the airbag bunch based model;
- damping below the dashed lines;
- realistic beam pipe, ΔQ_{coh} , ΔQ_{sc} ramps of the SIS100 bunches.