SPACE CHARGE EFFECTS IN FFAGS:

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Overview:

- Quick reminder: concept of FFAG accelerator.
- Parametric study to allow the study of the space charge effects: idea and main findings.
- Space charge effects in scaling/non-scaling FFAG.
- Conclusion.

Introduction

- Question1: In order to understand space charge effects in FFAG, it is important to carry out parametric studies ⇨ the phase advance per cell is the natural parametrization because it allows the studies of resonance effects.

- Question2: Can we restore the scaling property of a scaling FFAG even in presence of space charge effects?

Reminder: scaling FFAG

$$\begin{cases} \dfrac{d^2x}{d\theta^2} + \left(\dfrac{R^2}{\rho^2(R,\theta)}[1-n]\right)x = 0 \\ \dfrac{d^2y}{d\theta^2} + \left(\dfrac{R^2}{\rho^2(R,\theta)}n\right)y = 0 \end{cases}$$

- In the scaling FFAG concept, the two cardinal conditions to ensure the tunes are constant in both planes are:

$$\left.\frac{\partial n}{\partial p}\right|_{\theta=const} = 0$$

$$\left.\frac{\partial}{\partial p}\left(\frac{R}{\rho}\right)\right|_{\theta=const} = 0 \quad \Rightarrow \quad B = B_0\left(\frac{R}{R_0}\right)^k \times F(\theta)$$

- It was shown by Symon that these two conditions are sufficient and yield:

$\nu_{x_0}^2 \approx k+1$ and $\nu_{y_0}^2 \approx -k + \mathcal{F}^2$

<u>Question3</u>: Are they necessary as well?

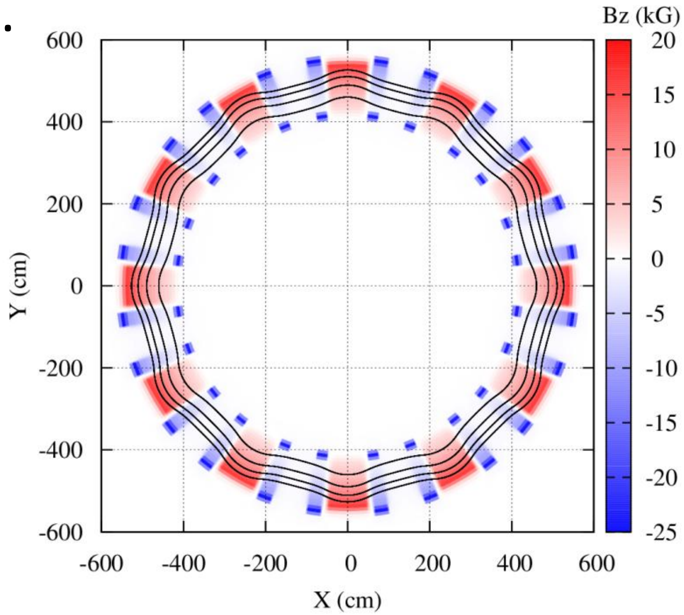
Example of a DFD triplet

- An extension of the mean field index k as defined by Symon consists in introducing its azimuthal variation in the following way:

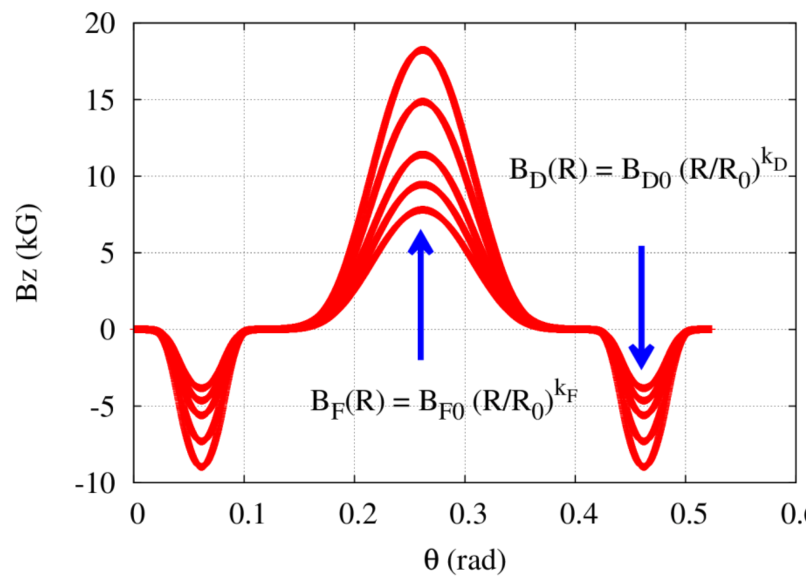
$$k_i = \frac{R}{B_i}\frac{dB_i}{dR} \quad ; \quad i = F, D, drift$$

where B_i is the vertical component of the field averaged over the width of the element.

Particularly useful for non-linear magnets or when n changes along the azimuth.



Example of several closed orbits: the lattice consists of 12 DFD triplets.



Magnetic field along several closed orbits

Parametric study for FFAG lattice

1) We build the model by generating a median plane field map for a given (k_F, k_D). Tracking is performed using ZGOUBI: Median plane anti-symmetry is assumed and the Maxwell equations are accommodated which yields the Taylor expansions for the three components of the magnetic field.

2) Search for the closed orbits between injection and extraction energy.

3) For each closed orbit, the matching condition is calculated and tracked over several turns. One assumes a symmetric beam distribution. The number of betatron oscillations is then computed:

$$\nu_{x,y}^{m} = \langle \nu_{x,y} \rangle = \frac{1}{NCO} \sum_{i=1}^{NCO} \nu_{x,y,i}$$

$$\nu_{x,y}^{rms} = \langle \nu_{x,y}^{2} \rangle^{1/2} = \left(\frac{1}{NCO} \sum_{i=1}^{NCO} \left(\nu_{x,y,i} - \nu_{x,y}^{m} \right)^{2} \right)^{1/2}$$

Parametric study for FFAG lattice

4) Repeat the same steps with and without space charge. This provides the tune depression. Our numerical simulation consists of a frozen space charge model. We assume that the FFAG is operating in emittance-dominated regime. Therefore, the space charge is treated as a small perturbation.

NB: The closed orbits formalism that we use for our analysis is mainly valid under the assumption that all orbits are slowly changing with time, i.e the acceleration rate is small enough that the damping of the betatron oscillations can be considered adiabatic.

Study of the undepressed tunes

(4 slides)

Bogoliubov method of averages to compute the tunes

- It is found that the Symon formula does not hold in the general case where $k_F \neq k_D$ since the tune changes with the energy.

- Using the BKM's method of averages, one can compute approximately the frequencies of the betatron oscillations and their dependence on the average field index of the F and D magnet. One obtains:

$$\boxed{\nu_x^2(R_E) = \sum_i \beta_i(R_E) - \sum_i \alpha_i(R_E) \times k_i(R_E)}$$

$$\boxed{\nu_y^2(R_E) = \sum_i \alpha_i(R_E) \times k_i(R_E) + \mathcal{F}^2 \left[1 + 2\tan^2(\xi)\right]}$$

where $\alpha_i(R_E) = \frac{-1}{2\pi/N}\int_{\theta_i}\mu(R,\theta)d\theta = \frac{-1}{2\pi/N}\int_{\theta_i}\frac{R}{\rho}d\theta$ $\quad \beta_i(R_E) = \frac{1}{2\pi/N}\int_{\theta_i}\mu(R,\theta)^2 d\theta = \frac{1}{2\pi/N}\int_{\theta_i}\left(\frac{R}{\rho}\right)^2 d\theta$

If $\boldsymbol{\kappa = k_F - k_D \neq 0}$, the tunes are energy-dependent and the orbits are not similar
⇨ Non scaling FFAG.

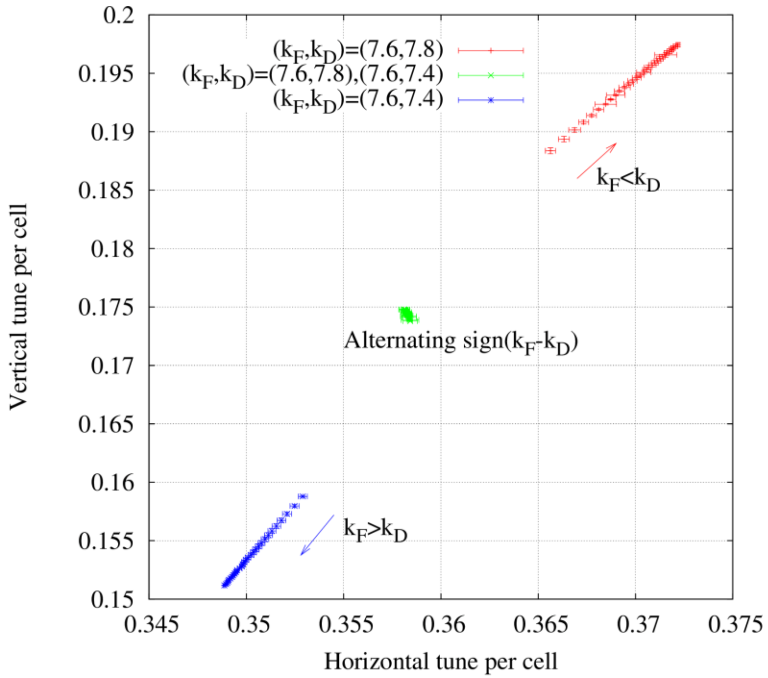
Bare tune excursion

- It is found that two regimes can be distinguished depending on the sign of $\boldsymbol{\kappa = k_D - k_F}$:

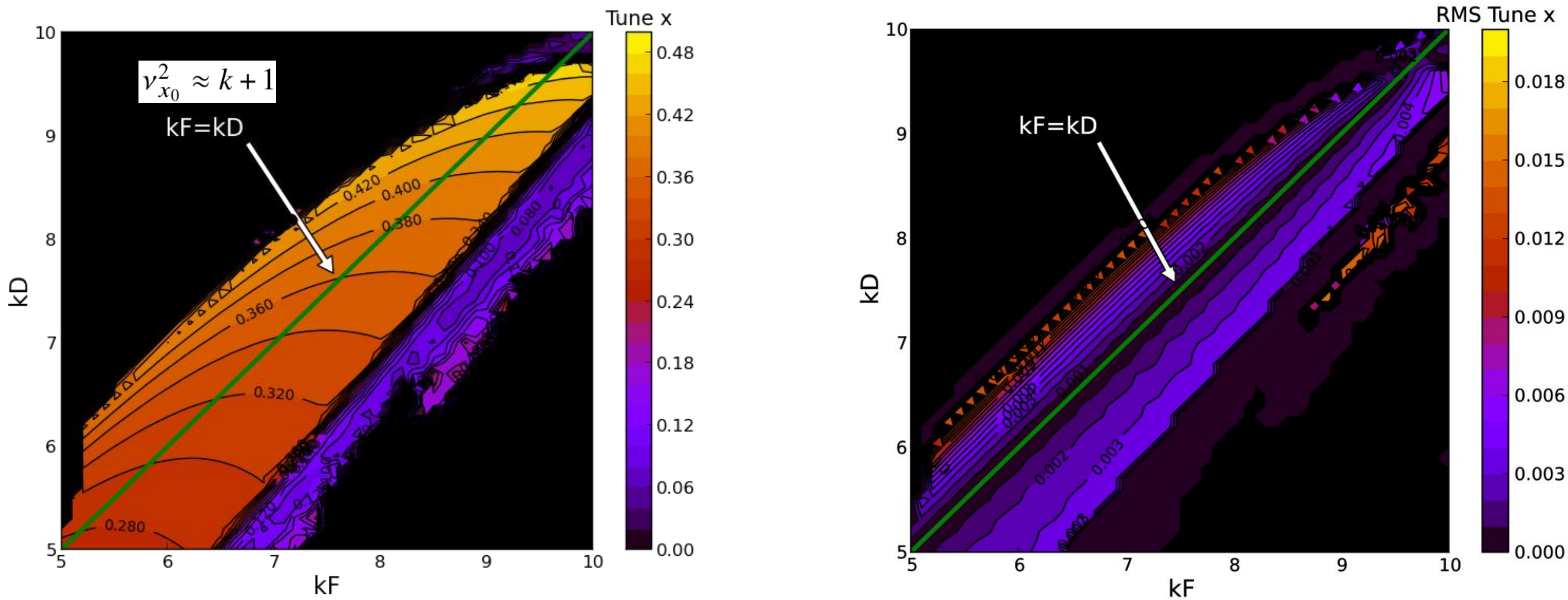
• If $\boldsymbol{\kappa > 0}$ then the phase advance per cell is an increasing function of the energy in both planes.

• If $\boldsymbol{\kappa < 0}$ then the phase advance per cell is a decreasing function of the energy in both planes.

⇨Combining the two effects in a single FFAG lattice, i.e alternating $\boldsymbol{\kappa}$, allows to obtain a fixed tune FFAG. This demonstrates that the two cardinal conditions are **sufficient but non-necessary** conditions for a fixed tune FFAG.



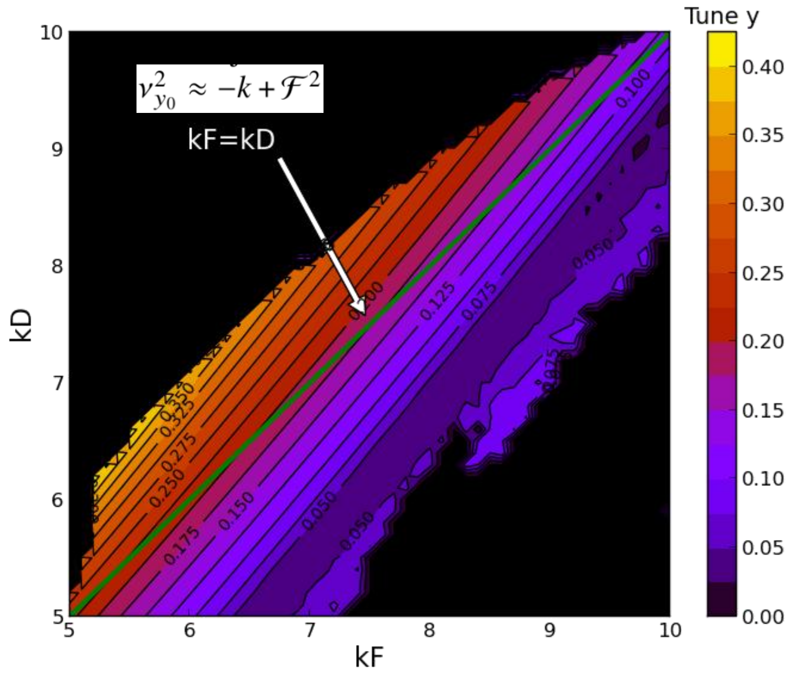
Stability diagrams for FFAG: <u>horizontal plane</u>

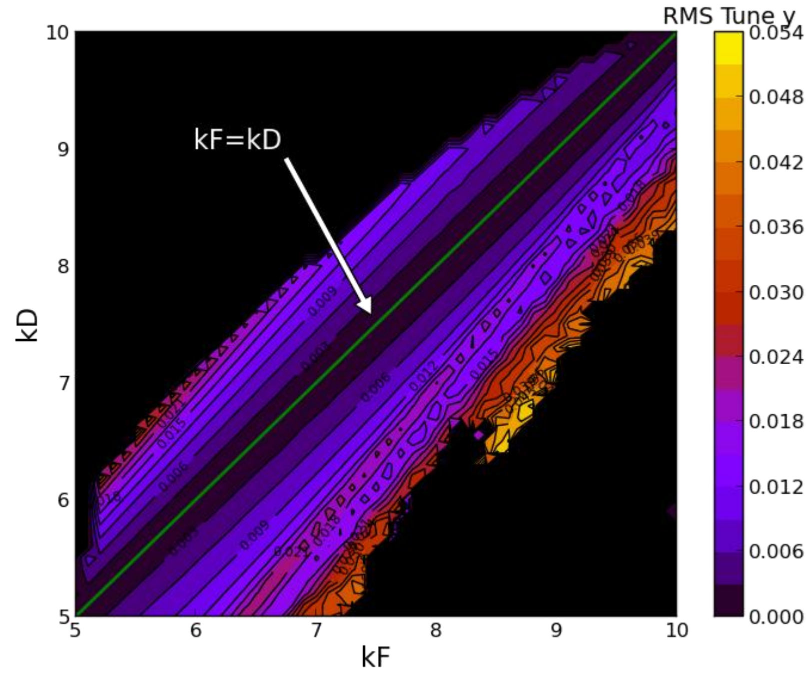


In the vicinity of the central line, i.e $k_F = k_D$, the Symon formula is qualitatively verified.

One can also observe that, for large k values, increasing $|\boldsymbol{\kappa}|$ makes the orbits quickly unstable, thus the stability diagram shrinks.

Stability diagrams for FFAG: vertical plane





A key finding of this study is that, in the vicinity of $\boldsymbol{\kappa} = \mathbf{0}$, the RMS tunes are proportional to $|\boldsymbol{\kappa}|$ in both planes :

$$\nu_{x,y}^{rms} \approx a_{x,y}|k_F - k_D| = a_{x,y}|\kappa|$$

Study of the depressed tunes

(7 slides)

Case of scaling FFAG

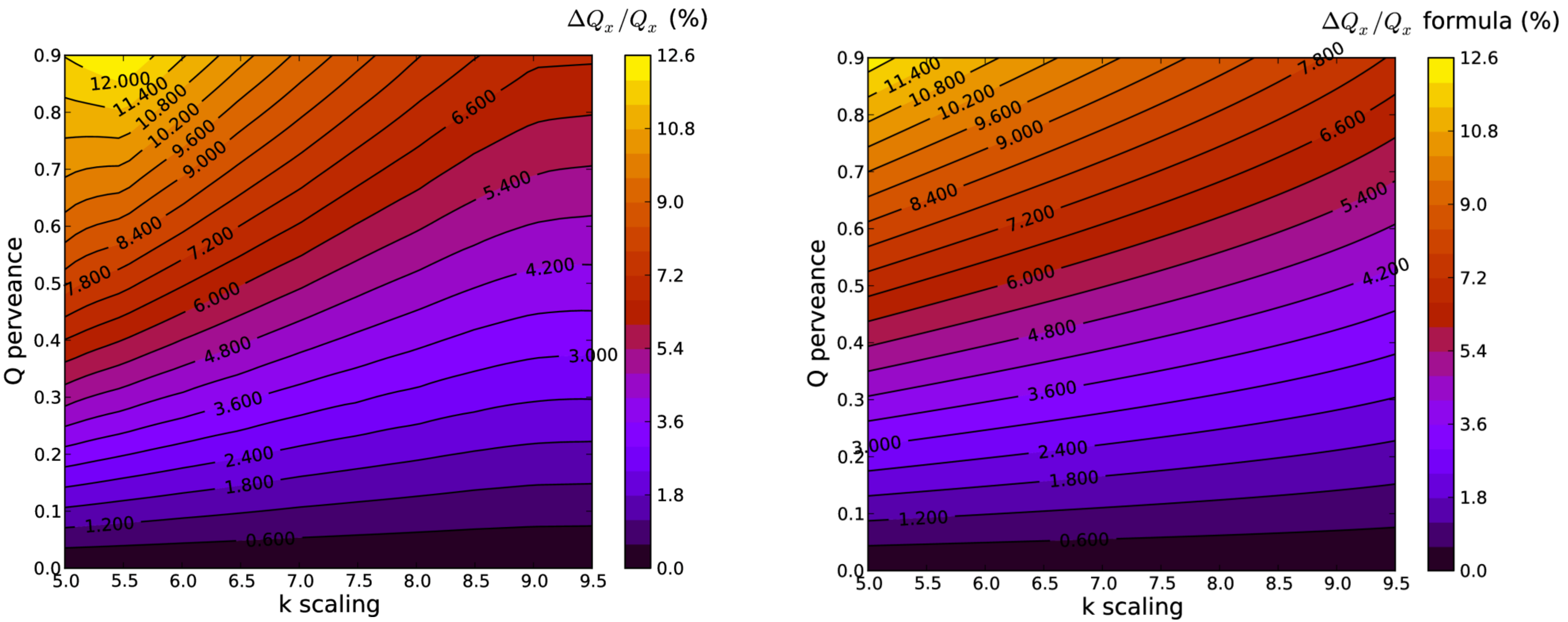
We assume that $\boldsymbol{\kappa = 0}$, thus the undepressed phase advance per cell is constant. We also assume a KV beam distribution and a symmetric beam.

The procedure described above is applied by varying the average field index k of the magnets as well as the linear charge density λ thus the perveance Q.

It can be shown that:

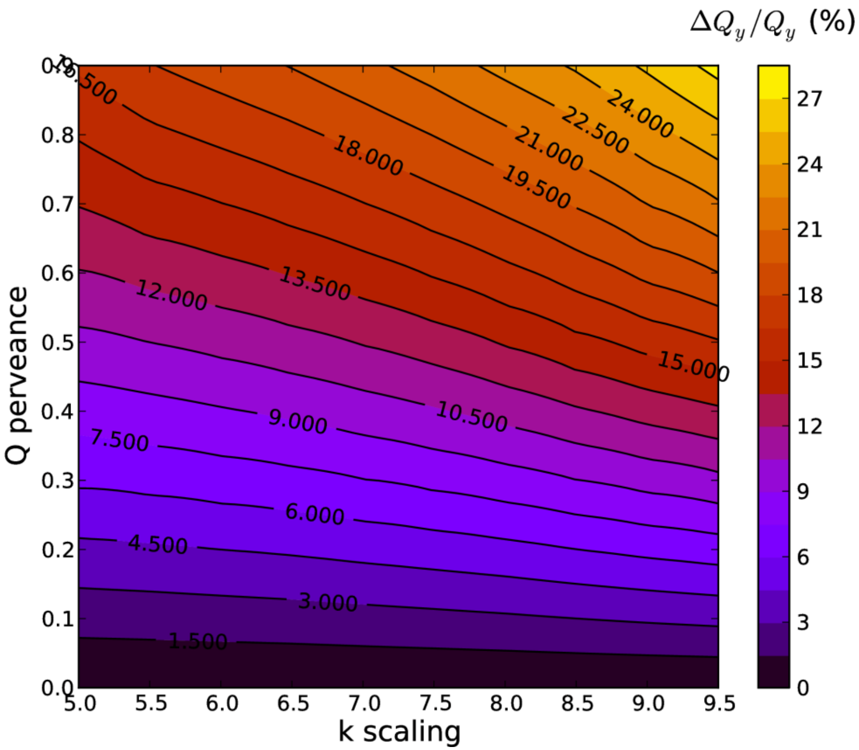
$$\frac{\Delta\nu_x}{\nu_{x_0}} \approx \frac{RQ}{\epsilon} \frac{(k+1)^{-3/4}}{(k+1)^{-1/4} + (-k + \mathcal{F}^2)^{-1/4}} \approx f(k, Q)$$

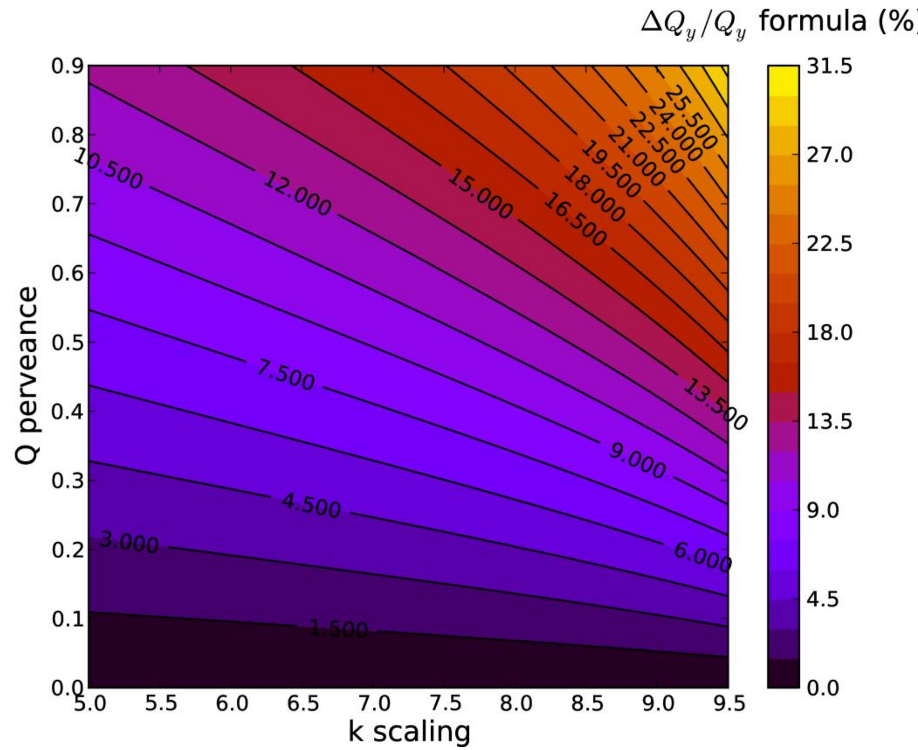
Case of scaling FFAG



Good agreement in the horizontal plane.

Case of scaling FFAG





Good agreement in the vertical plane.

However, the tune excursion changes considerably and $\Delta\nu_{x,y} \propto \frac{R}{\beta\gamma^2}$

Can we find an FFAG lattice for which the depressed tune remains constant?

Compensation scheme

The idea is to play either with k_F or k_D in such a way as to counteract the space charge effects which induce a larger tune shift at the injection.

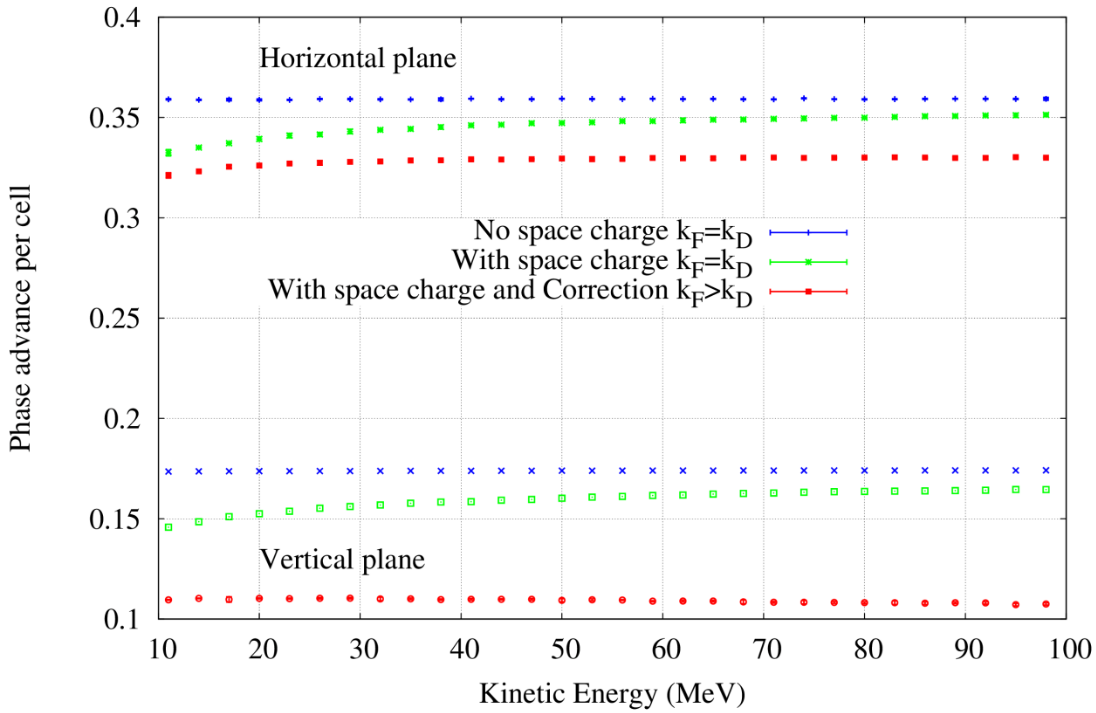
As shown earlier, if $\kappa < 0$, this condition can be satisfied since the bare tune is a decreasing function of the energy in both planes. The problem writes in the following way:

$$\begin{cases} \nu_x = \nu_{x0} + \delta\nu_x(\delta k_F, \delta k_D) + \delta\nu_x(space\ charge) \\ \nu_y = \nu_{y0} + \delta\nu_y(\delta k_F, \delta k_D) + \delta\nu_y(space\ charge) \end{cases}$$

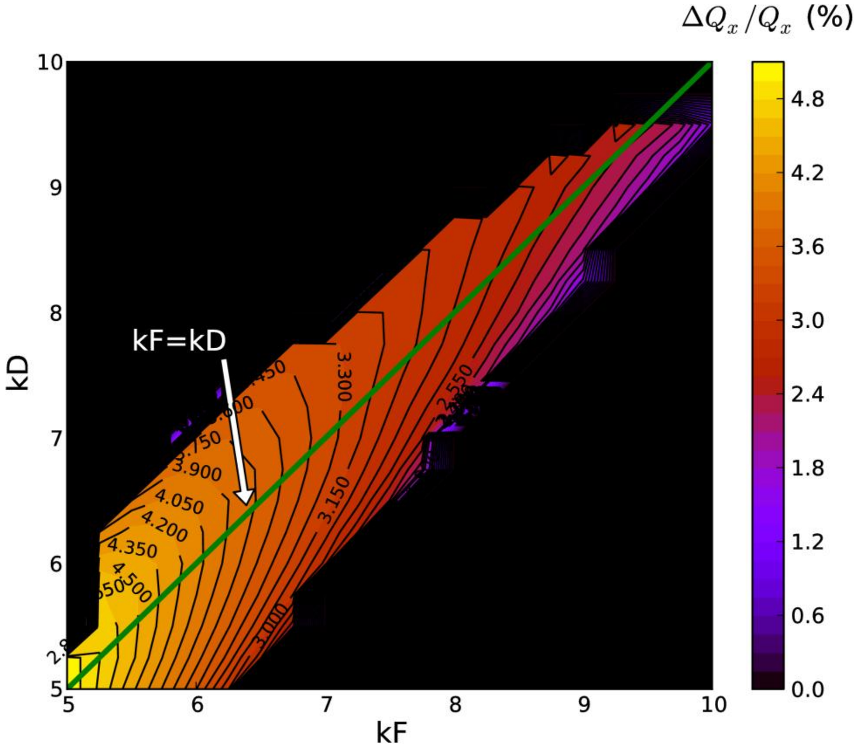
Compensation scheme

The problem can be solved by equating the RMS tune excursion of the KV beam with the RMS tune excursion of the bare tunes. Solving for κ ,this yields:

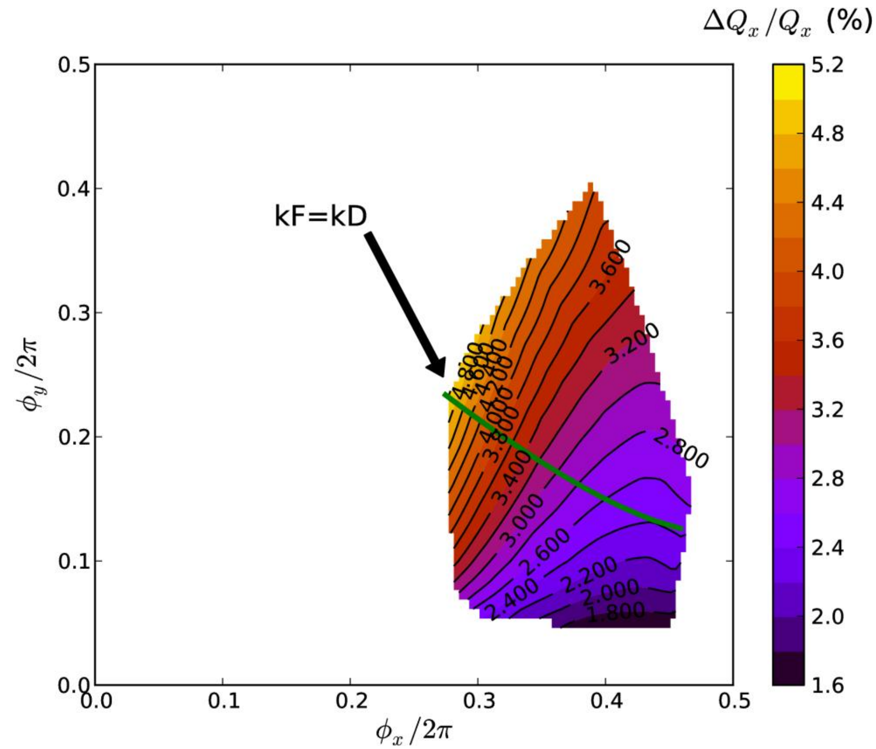
$$\kappa \in \left[\frac{\nu_x^{rms}(sc)}{a_x} : \frac{\nu_y^{rms}(sc)}{a_y} \right]$$



Stability diagram for Non-scaling FFAG



(k_F, k_D) **diagram.**



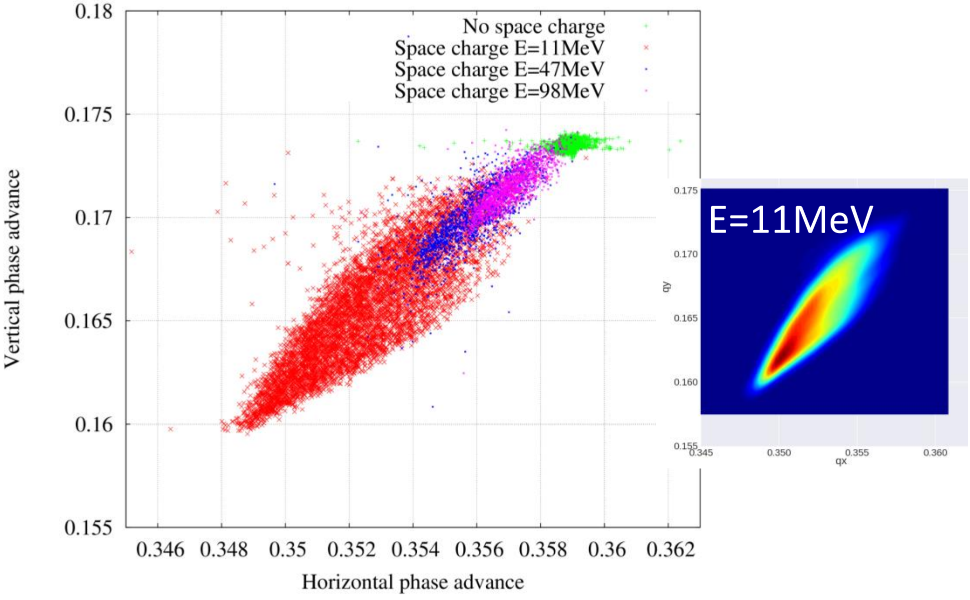
Phase advance diagram.

The core of the distribution can be positioned in the tune diagram in a way to avoid harmful resonances.

Tune spread

It is important to note that for a Gaussian distribution, the situation is more complicated due to the large tune spread.

The previous approach does not remediate the large tune spread of the beam.
However it remains valid for the core of the distribution which can be positioned in the tune diagram in a way to avoid harmful resonances.



Conclusion and Future plans

- Trim coils are necessary for this type of accelerators, especially for high intensity operation.

This should be considered in the preliminary design phase.

- Future plans include the studies of the resonance effects on the non-linear beam dynamics of FFAGs.

- How to minimize the space charge tune spread?

Thank you!