Collective Beam Instability and Beam Halo Due to Space Charge

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Outline

1. **Overview**
2. **Collective modes -- structure resonances**
3. **Halo mechanism – resonance between particle and collective mode**
4. **Summary**

In this talk we consider the space charge effect in rms matched beams only. Rms mismatched beams are beyond of the scope of this talk.

Models in space charge dynamics research

1. **Rms envelope model**
   - **Rms envelope equation with exact self-consistent KV distribution. (Sacherer, 1971).**
   - **rms equivalent, envelope model could be used with confidence (Sacherer, 1973).**
   - **Well-known envelope instability criteria is given $\sigma_0 < 90^\circ$ (J. Struckmeier, et. al, 1984).**
   - **Nonlinear resonance, chaos sea, bifurcation in 1.5D envelope dynamics with mismatch.**
   - **Test-particle model to interpret beam halo mechanism and halo suppression with multi-poles.**
2. **Self-consistent Vlasov model**
   - **Solve the perturbed Vlasov-Poisson equation simultaneously (R. L. Glusktern, 1970).**
   - **General solution of the arbitrary order of mode in periodical channel for isotropic beam is expressed as the form of Jacobi Matrix—structure resonances stop bands (I. Hofmann, et. al, 1983; Chao Li, 2016).**
   - **Arbitrary orders of collective modes in smooth channel for anisotropic beam is given (I. Hofmann, 1998).**
3. **Numerical model**
   - **PIC code – TOPO and P-TOPO (Chao Li, Zhicong Liu, Yaliang Zhao, …)**

Techniques used in TOPO:

- Under development;
- T-code;
- Moving cuboid meshes for space charge calculating;
- First weighting methods;
- Symplectic integrator;
- Poisson solver with FFT;
- Lorentz transformation (electric fields are multiplied by the factor of $1/\gamma^2$);
- P-TOPO is used in ADS injector I study (OpenMP).

2. Structure resonances – unstable collective modes.

- **Vlasov equation, $f=f_0+f_1$, suppose f_0 is the KV beam profile—classical perturbation theory.**
- **Solve the Vlasov-Poisson equation simultaneously, self-consistent process**
- **Construct the integral of the discontinuity of the surface electric filed $I_{j;k,l}$**
- **Collective instability is decided by the Jacobi of $I_{j;k,l}$**
- **Isotropic beam and periodic channel-structure resonance**
- **Low orders modes are naturedly included in higher order**
- **Nonlinear perturbation is both the resonance driving force and damping term.**
- **Results obtained from KV as a guidance.**

$$\frac{Df_1}{Ds} = \{\frac{\partial}{\partial s} + \frac{1}{\beta_x}[p_x \frac{\partial}{\partial x} - x\frac{\partial}{\partial p_x}] + \frac{1}{\beta_y}[p_y \frac{\partial}{\partial y} - y\frac{\partial}{\partial p_y}]\} f_1$$

$$= 2\frac{N}{\pi^2 \epsilon}[p_x \frac{\partial V}{\partial x} + p_y \frac{\partial V}{\partial y}]\delta'(x^2 + p_x^2 + y^2 + p_y^2 - 1).$$

$$\Delta V_n = -\iint f_1 dp \quad V_n = \sum_{m=0}^{n} A_m(s) x^{n-m} y^m + \sum_{m=0}^{n-2} A_m^{(1)}(s) x^{n-m} y^m + \cdots$$

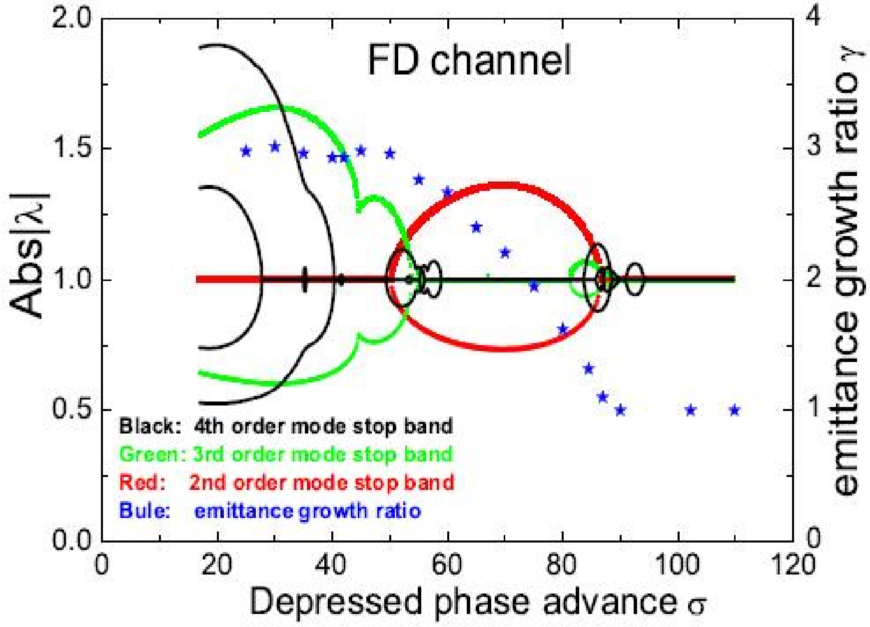
$$I_{j;k,l} = \int_0^s A_j(s') \sin[k(\psi_{x'} - \psi_x) - l(\psi_{y'} - \psi_y)] ds';$$

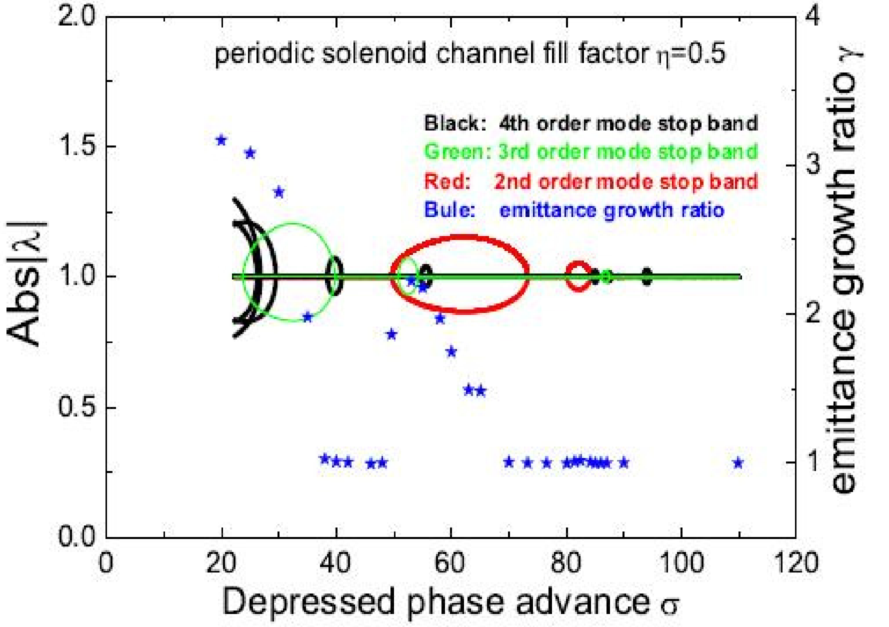
$$\frac{1}{C_{k,l}(s)} \frac{d}{ds}[\frac{1}{C_{k,l}(s)} \frac{dI_{j;k,l}}{ds}] + I_{j;k,l} = -\frac{1}{C_{k,l}(s)} A_j.$$

I. Hofmann, L. J. Laslett, L. Smith, and I. Haber, Part. Accel. 13, 145 (1983).
Chao Li, R, A, Jameson, Qing Qin, Collective mode study, to be published (2016).
Ref. R. Dilao, *Nonlinear dynamics in particle accelerators*, Vol. 23 (World Scientific, 1996)

Beam reaction to the structure resonance--numerical results with code TOPO

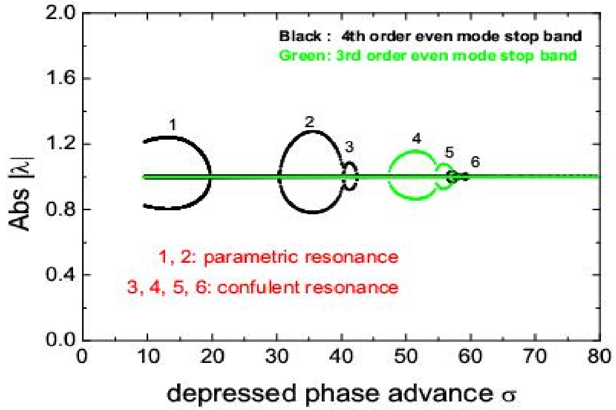
- **Emittance growth ratio in 200 periods, eigenvalues of the 2nd, 3rd and the 4th order modes in FD and periodic solenoid channel when $\sigma 0 = 110$, KV initial beam.**



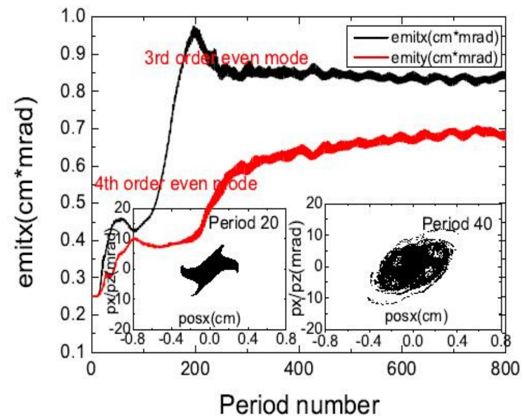


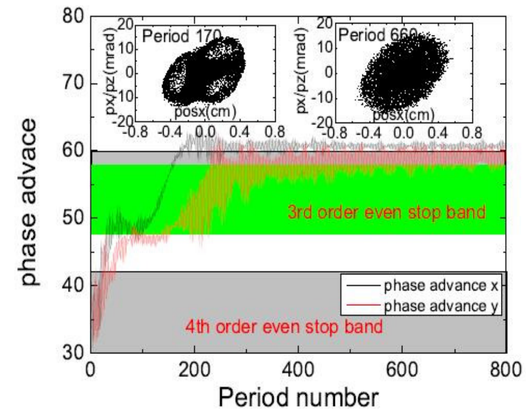
- **The broadened collective stop bands predict well the areas where the rms emittance growth take place.**

beam moves out of the stop band spontaneously--4th / 3rd order mode: $4\sigma \sim 180$ / $3\sigma \sim 180$

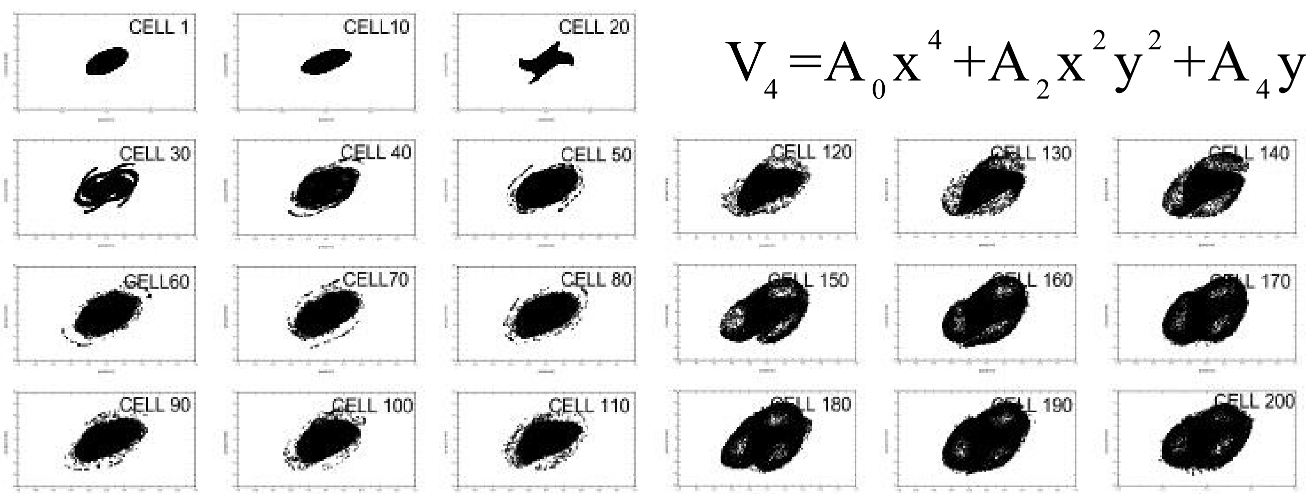


Stop bands pitches $\sigma 0 = 80$





Emittance and instantaneous phase advance
With initial KV beam, $\sigma 0 = 80$, $\sigma = 35$



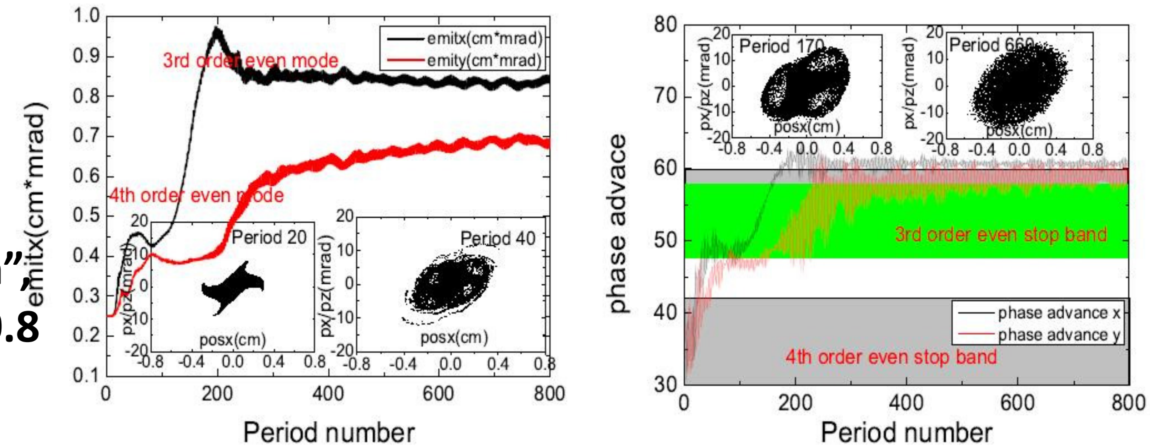
$$V_4 = A_0 x^4 + A_2 x^2 y^2 + A_4 y^4 + A_0^{(1)} x^2 + A_2^{(1)} y^2$$

$\sigma 0 = 80$
$\sigma \sim 35$
FD

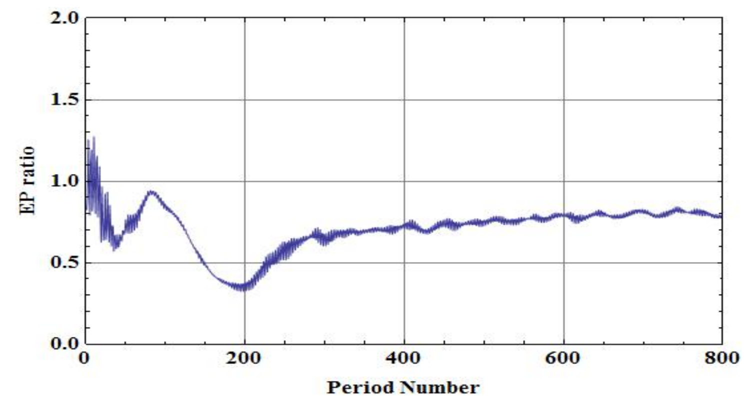
x-px phase space evolution along the FD channel

beam moves out of the stop band spontaneously--4th / 3rd order mode: $4\sigma \sim 180$ / $3\sigma \sim 180$

- **KV initial symmetric beam**
- **Initial rms EP condition**
- **Backwards to EP in first 300 periods**
- **Get to a final "local equilibrium", that stays at EP ration around 0.8 for a long time.**
- **Towards EP by the influence of the higher order structure resonance and anisotropic resonances in a long time scale.**
- **N-fold phase space structure are washed out.**
- **Local – description other than the steady state description.**



Emittance and instantaneous phase advance
With initial KV beam, $\sigma 0 = 80$, $\sigma = 35$, 800 periods



EP ratio with initial KV beam, $\sigma 0 = 80$, $\sigma = 35$, 800 periods.

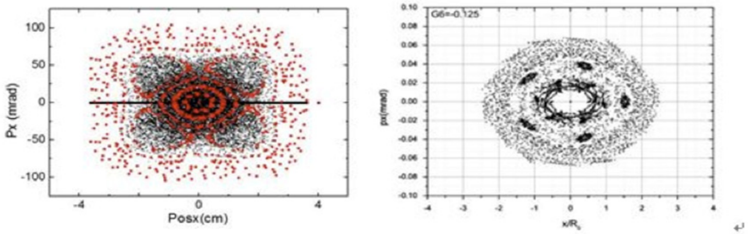
Discussion on structure resonance

- **Verify the effect of collective modes**
- **2nd order (envelope instability) and 4th order resonance.**
- **Effect nth order collective modes**
- **Resonance and damping**
- **Local parameter matters--beyond the discussion base on steady state**
- **“Local equilibrium” state in EP or not.**
- **N-fold phase space– beam halo: side effect of structure resonance**

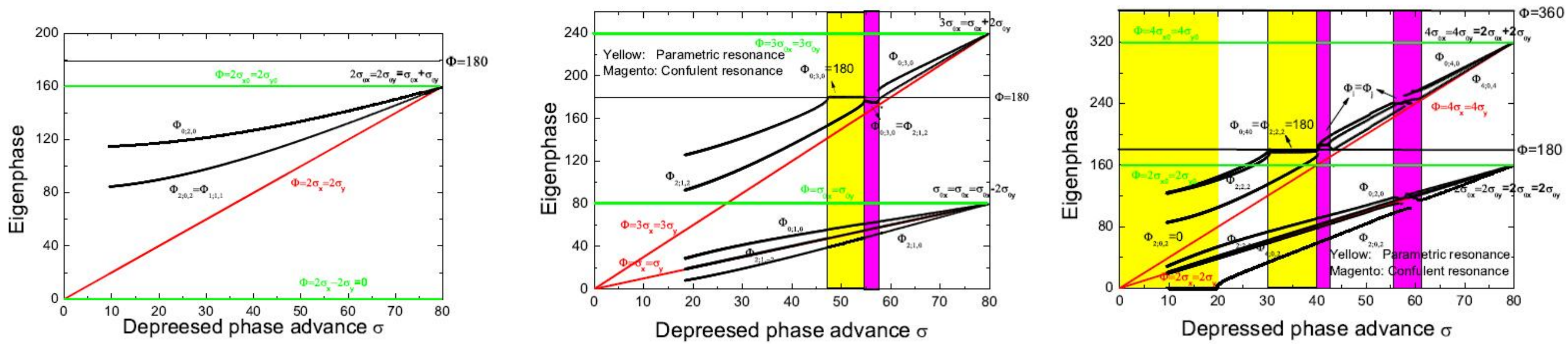
3. Halo mechanism – resonance between particle and collective mode

“frequencies” in rms matched beam

1. **Envelope oscillation phase advance: $\phi_e = 360$**
2. **Single particle phase adance: σ_s**
3. **Collective mode eigenphase: $\Phi_{j;k,l}$**



1. **The nth order of collective parametric resonance: $\Phi_{j;k,l}/\phi_e$=m/2n**
   - **Lead to collective mode instability. ->NO!**
2. **The nth order of single particle–core resonance: σ_s/ϕ_e=m/n**
   - **Might lead to beam halo, but in a long time scale -> NO!**
3. **The nth order of single particle–collective mode resonance: $\sigma_s/\Phi_{j;k,l}$ = m/n**
   - **Trapped collective modes give sustained kick to particle to go into beam halo.**



Halo mechanism – resonance between particle and collective mode

The perturbed potential $$V_n = \sum_{m=0}^{n} A_m(s) x^{n-m} y^m$$

Single particle Hamiltonian $$H(x, p_x, y, p_y; s) = \frac{1}{2}(K_x(s)x^2 + p_x^2) + \frac{1}{2}(K_y(s)y^2 + p_y^2) + V_n(x, y; s)$$

Action – Angle frame $$H(\Phi_x, J_x, \Phi_y, J_y; \theta) = \nu_x J_x + \nu_y J_y + \sum_{p,k} A_{p,k}(\theta)\cos(p\Phi_x + k\Phi_y)$$

In Fourier expression $$V_n = \sum_{p,k} V_{p,k}(\theta) = \sum_{p,k}\sum_{l} G_{p,k;l} e^{i(p\phi_x + k\phi_y - l\theta + p\chi_x - k\chi_y)}$$

Resonance condition: *p+k=l*

The resonance condition is *p+k=l*, related resonance strength is given by $G_{p;k,l}$. In this case, perturbed space charge potential V_n is treated as external field. Resonance condition is the same as that in circular machine.

Halo mechanism – resonance between particle and collective mode

In the parametric resonance stop band, the perturbed mode also oscillating with a phase shift Φ^e in one period.

$$\Phi^e = n*180$$

The perturbed potential is modified as

$$V_n = \sum_{p,k} V_{p,k}\kappa(\theta)e^{i\Phi^e}$$

$$= \sum_{p,k}\sum_{l} g_{p,k;l} e^{i(p\phi_x + k\phi_y - l\theta + \Phi^e + p\chi_x - k\chi_y)}$$

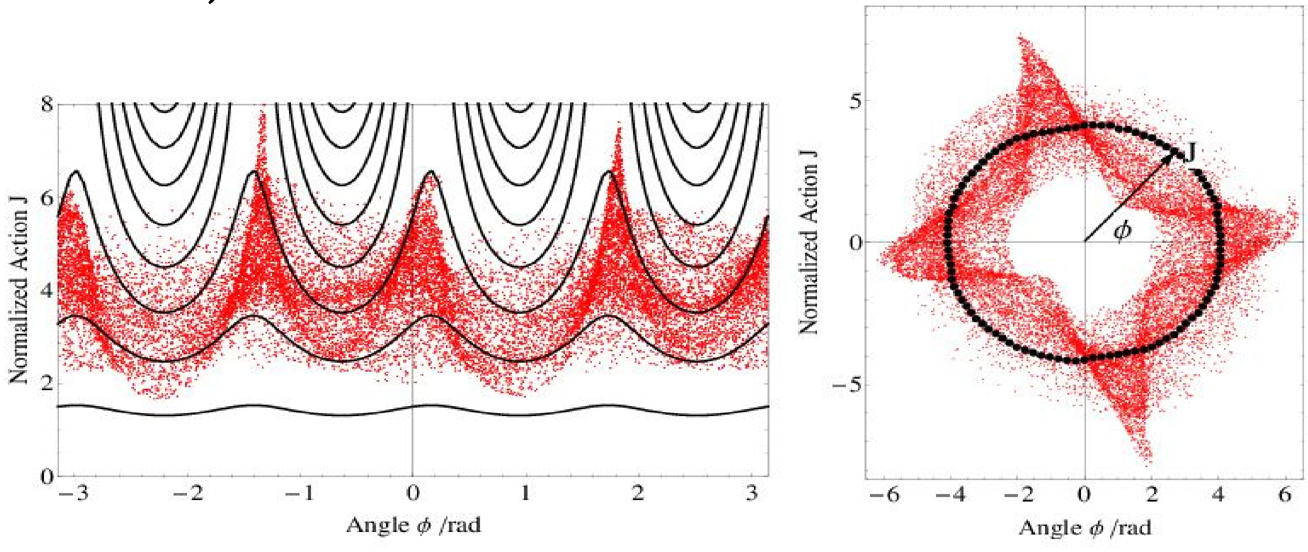
Resonance condition is modified as $p + k = l - 1/2$, when n=1.

Halo mechanism – resonance between single particle and collective mode

Example: 4th order resonance *p=4, k=0, l=1*

$$H(\Phi_x, J_x, \Phi_y, J_y; \theta) = \nu_x J_x + G_{4,0;l} \cos(4\phi_x - l\theta + \Phi_{0;4,0} + \xi_{4,0;l})$$

Period 20, $\sigma 0 = 80, \sigma \sim 35$



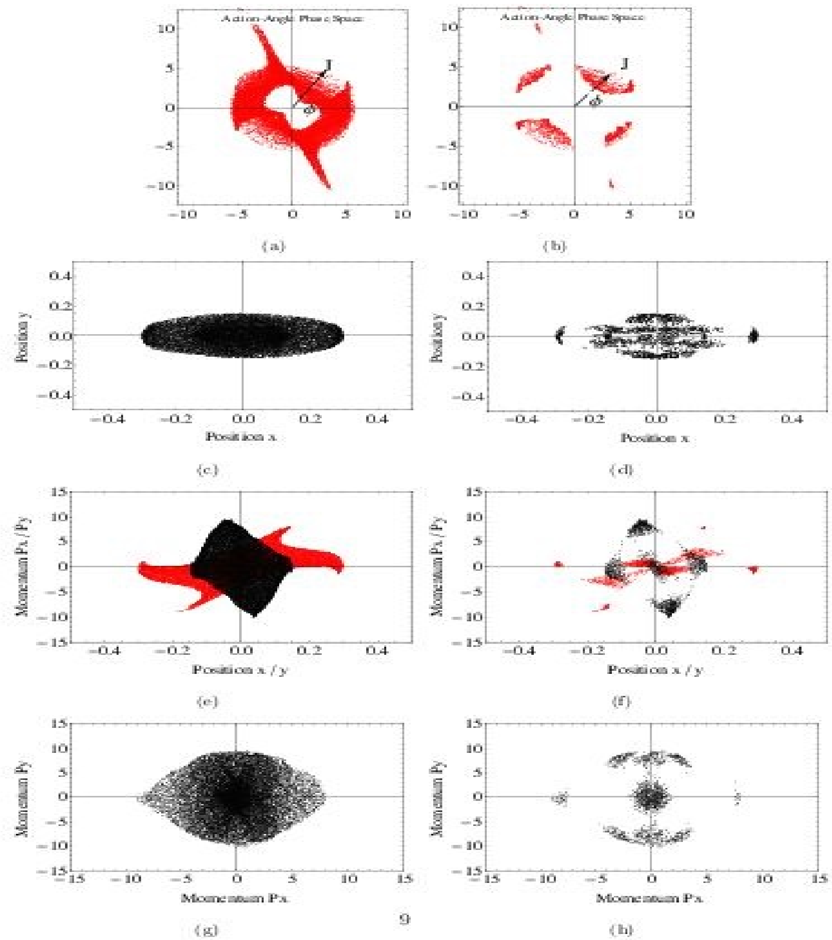
(a) (b)

Halo mechanism – resonance between single particle and collective mode

“Beam halo”: particles located in the fold structures in 4D action-angle space

Period 18, $\sigma 0 = 80, \sigma \sim 35$

- **The interaction between particle and collective mode well descried halo behavior.**
- **Halo could be defined as the particle whose Hamiltonian beyond certain separatrix.**
- **Halo could be hidden in the “core” in one projected plane (x-px), but they will be shown in another plane (y-py).**



Summary

- **Only internal mismatch**
- **Verify the effect of structure resonance (KV assumption as guidance)**
- **Nonlinearity is both resonance driving force and nonlinear damping source**
- **Resonance strongly depends on local parameters.**
- **Thermal model is not applicable—linacs are in local rms description, It is only applicable at the space charge limit**
- **“Beam halo”– resonance between single particle and collective mode**

Thanks for your attentions!