Use of RF Quadrupole Structures to Enhance Stability in Accelerator Rings

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Outline

- Introduction
- RF quadrupole for HL-LHC
- Synchro-betatron resonances
- Experimental studies in the SPS
- Summary

Introduction
Motivation

- **HL-LHC:** Increase the luminosity output of the LHC.
- **Key ingredients:** operate machine with higher bunch intensity and lower transverse beam sizes.
- Transverse collective instabilities may limit performance.
- Landau damping is a possible mitigation mechanism.
- **Main requirement:** incoherent betatron tune spread overlaps with the real part of the complex coherent tune shift $Re(\Delta Q_{coh})$ of the instability.

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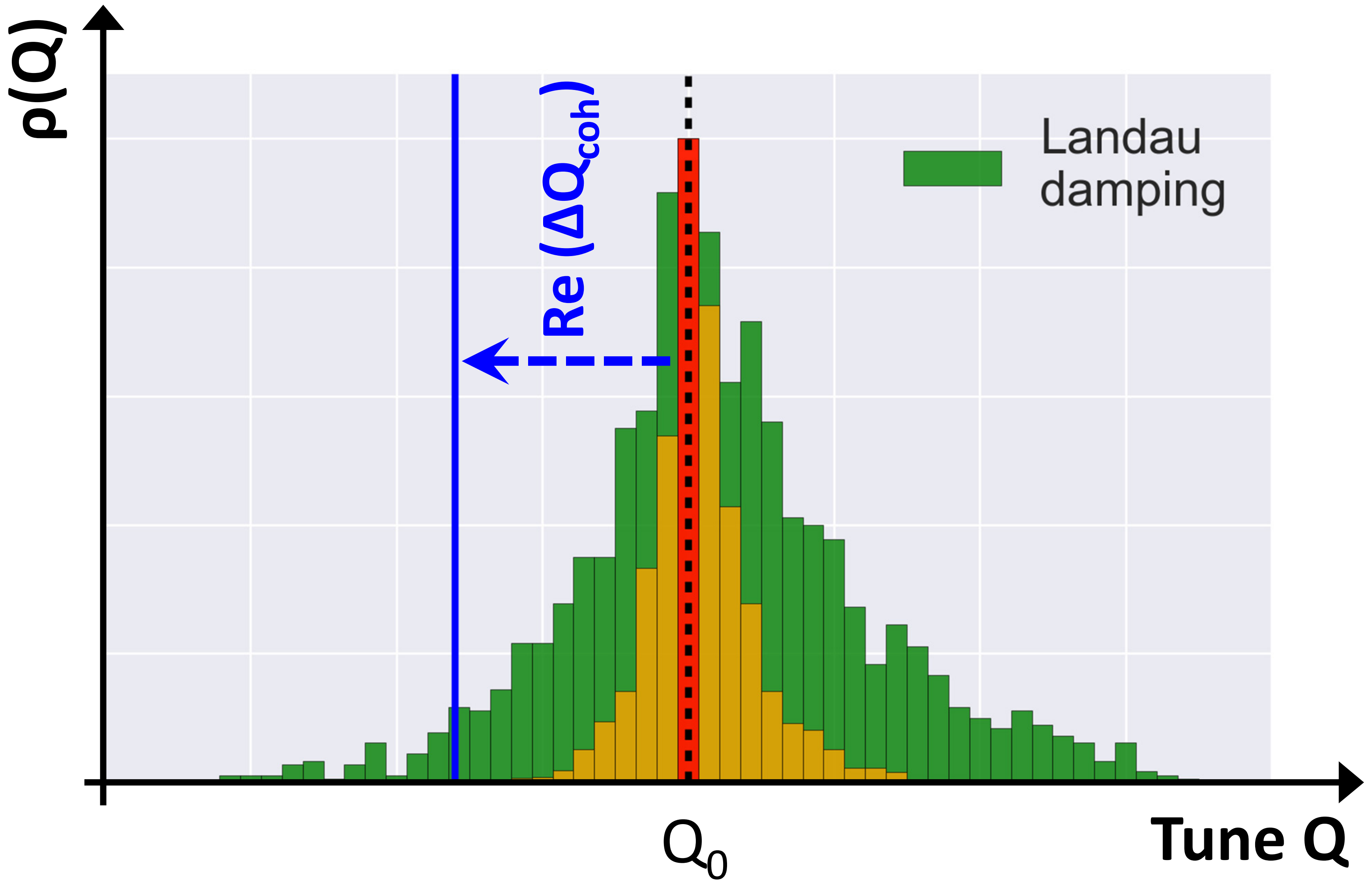


- In LHC, betatron detuning is successfully introduced by means of dedicated magnetic octupoles.
- However, they are often operated at their maximum strength to guarantee stable beams[16].
- In HL-LHC, detuning with *transverse* amplitude is reduced due to smaller transverse beam size.
- Additionally, the higher bunch intensity may cause more violent instabilities.

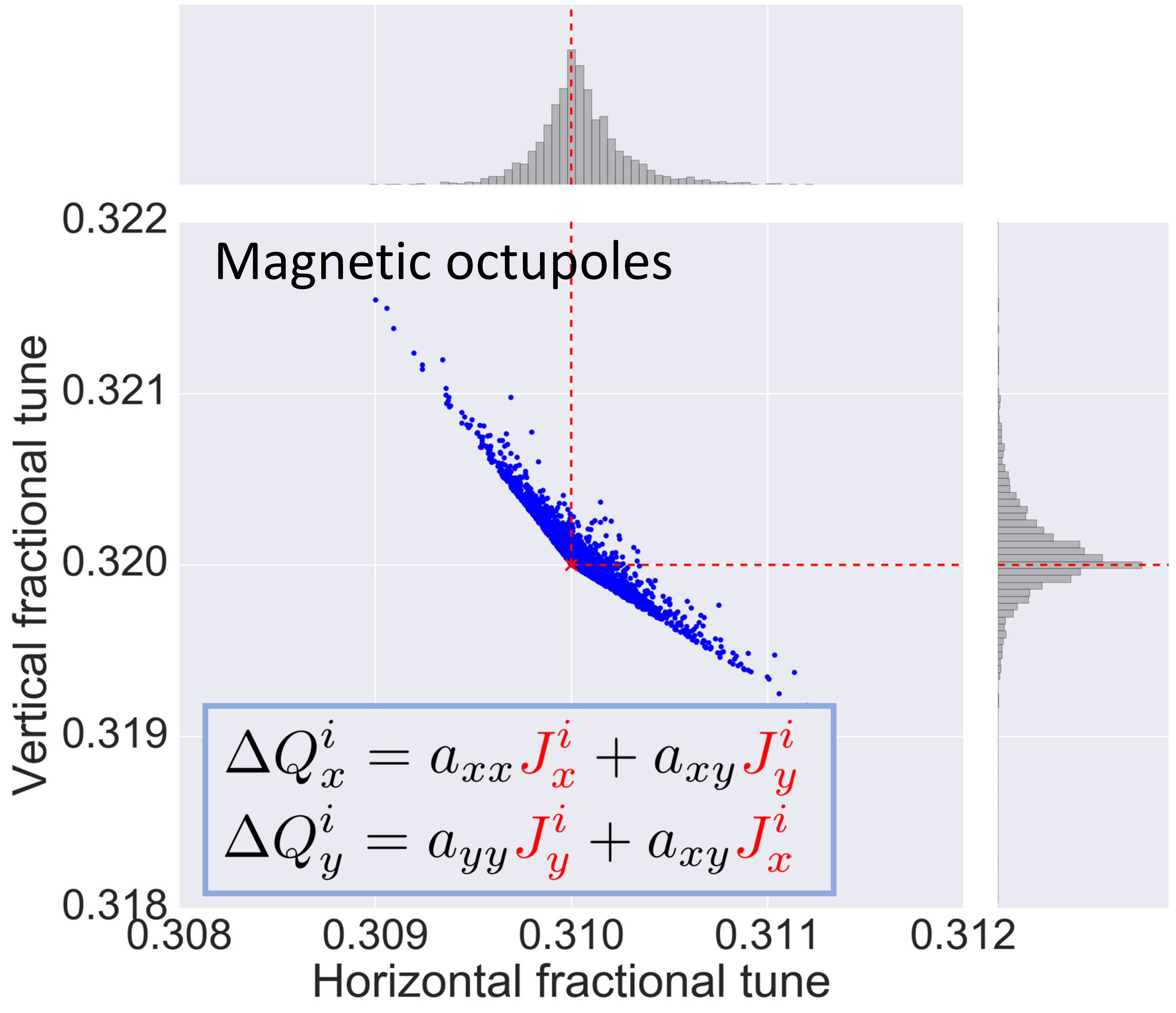
→ **Study possibility of betatron detuning with *longitudinal* amplitude for enhanced Landau damping in the transverse planes.**

Introduction

Betatron detuning with amplitude[5]

Transverse amplitude

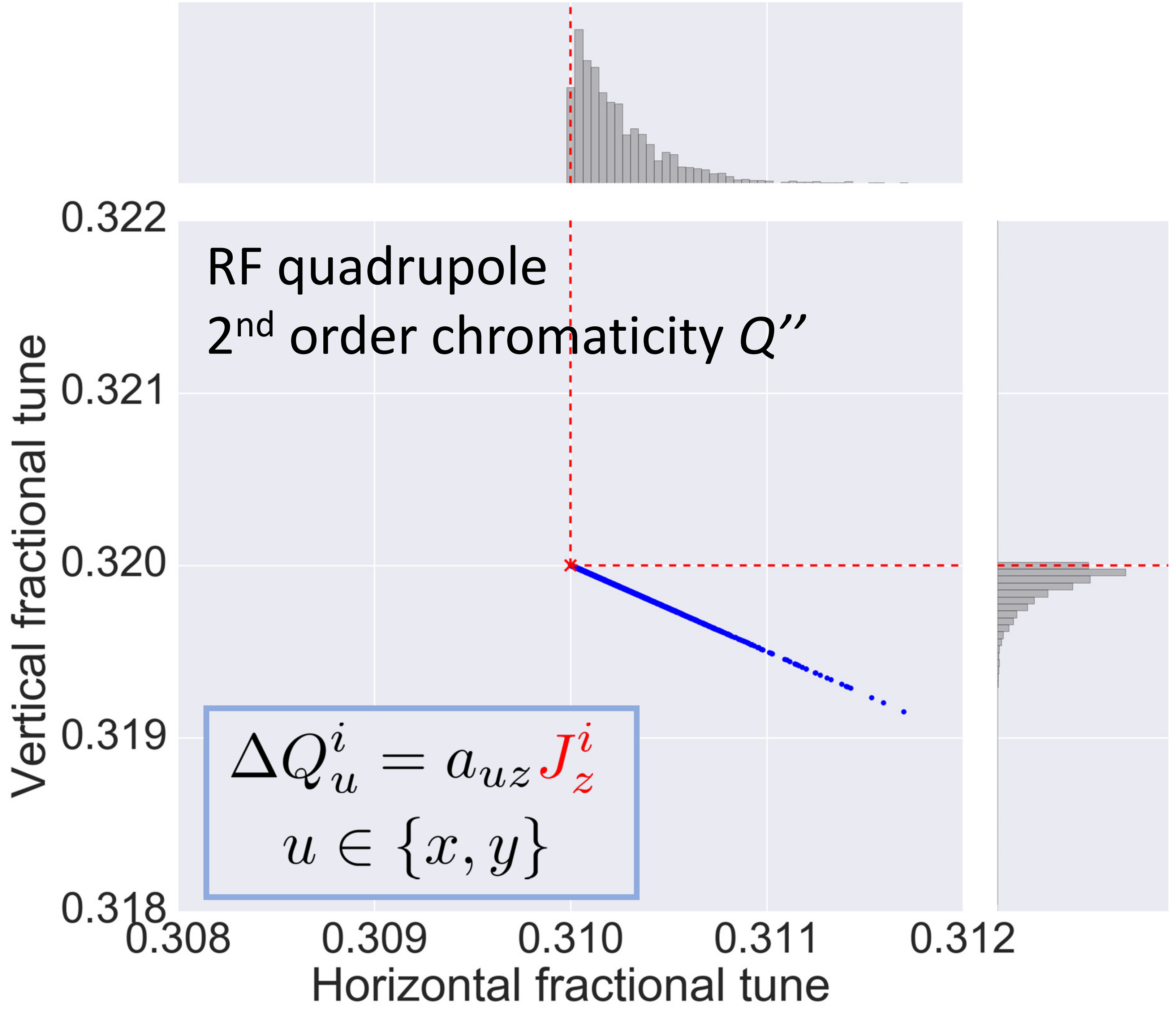
$$\Delta Q^i_{x,y} = \Delta Q^i_{x,y}\left(J^i_x, J^i_y\right)$$



Detuning strength is directly affected by the smaller transverse beam size (action spread).

Longitudinal amplitude

$$\Delta Q^i_{x,y} = \Delta Q^i_{x,y}\left(J^i_z\right)$$



Detuning is orders of magnitude more effective due to large ratio between longitudinal and transverse action spread (emittance)[2-4].

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RF quadrupole and second order chromaticity Q''[2-4]

(I) RF quadrupole detuning

RF-modulated quadrupole kick

(anti-)on-crest of rf wave for $z_i = 0$

$$\Delta \mathbf{p}_\perp^i(t) = q b^{(2)} \cdot [y_i(t)\,\mathbf{u}_y - x_i(t)\,\mathbf{u}_x] \cdot \cos\left(\omega \frac{z_i(t)}{\beta c}\right)$$

translates into a betatron detuning

$$\Delta Q_{x,y}^i(t) \propto \pm \cos\left(\frac{\omega}{\beta c} z_i(t)\right) = \pm\left[1 - \frac{1}{2}\left(\frac{\omega}{\beta c}\right)^2 z_i(t)^2 + \mathcal{O}(z_i(t)^4)\right]$$

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$\sigma_z \ll \beta c/\omega$

$$\langle \Delta Q_{x,y}^i \rangle_{T_s} \propto \pm\left[1 - \frac{1}{2}\left(\frac{\omega \sigma_z}{\beta c}\right)^2 J_z^i\right]$$

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(II) Chromatic detuning

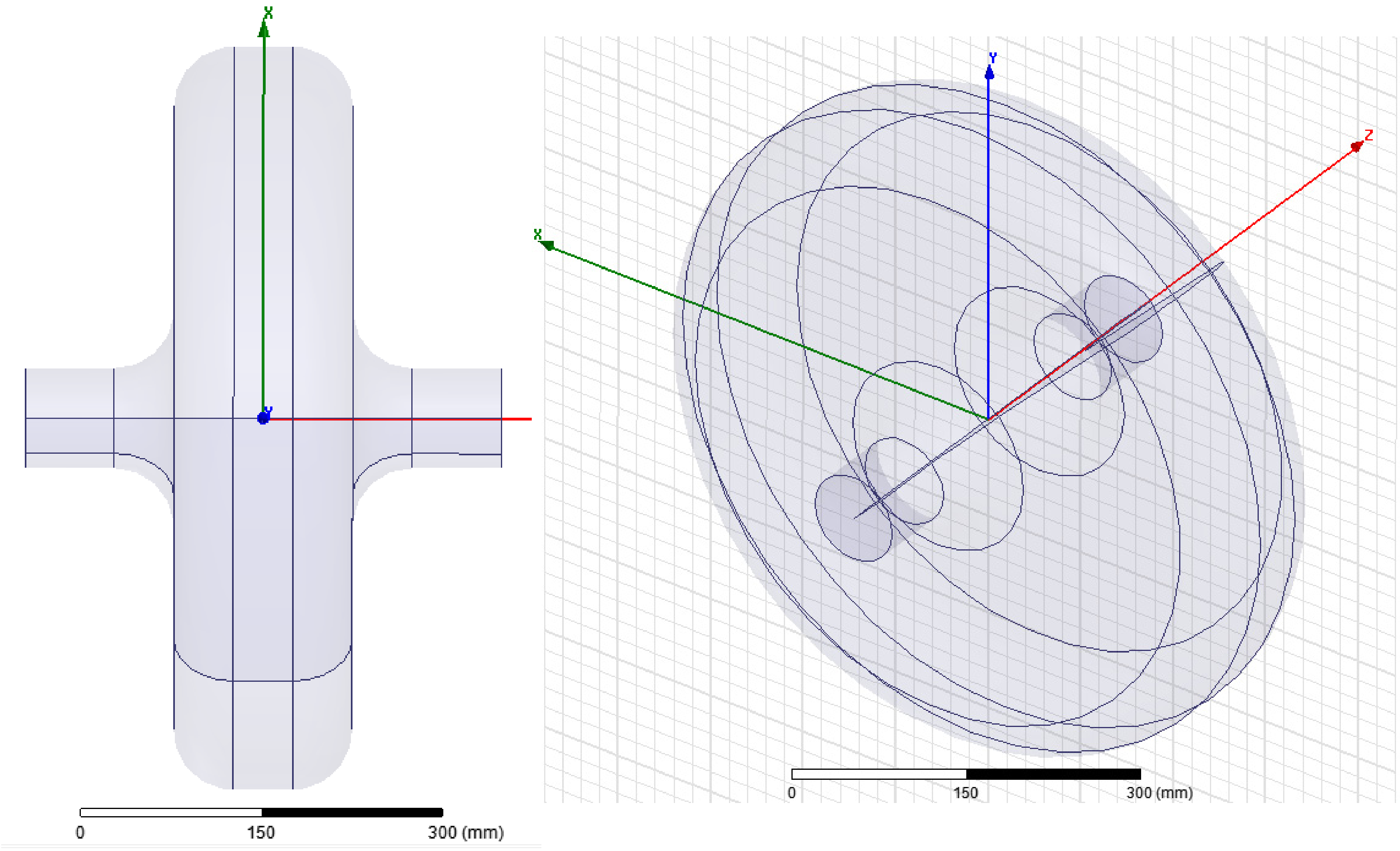
Betatron detuning from $\delta_i = \Delta p_i / p$

$$\Delta Q_{x,y}^{i}(t) = Q'_{x,y}\delta_i(t) + \frac{Q''_{x,y}}{2}\delta_i(t)^2 + \mathcal{O}\left(\delta_i(t)^3\right)$$

Second order chromaticity term leads equally to non-zero betatron detuning which resembles that of an RF quadrupole.

$$\langle \Delta Q_{x,y}^{i} \rangle_{T_s} = \frac{Q''_{x,y}}{2}\sigma_\delta^2 J_z^i$$

Example: *Pillbox cavity with TM_{210} mode.*



Not to be confused with the RFQ structure used in linacs!

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translates int

$\Delta Q^i_{x,y}(t) \propto \dots$

$\langle \Delta Q^i_{x,y} \rangle_{T_s} \propto \dots$

- **Betatron detuning indeed depends on the longitudinal action of the particle.**
- **The stabilising mechanisms introduced by the two methods are the same (first order) and can be studied simultaneously.**
- **This is important for experimental studies on a fast(er) time scale to benchmark the numerical model.**
- **Experiments with Q'' will be addressed later.**

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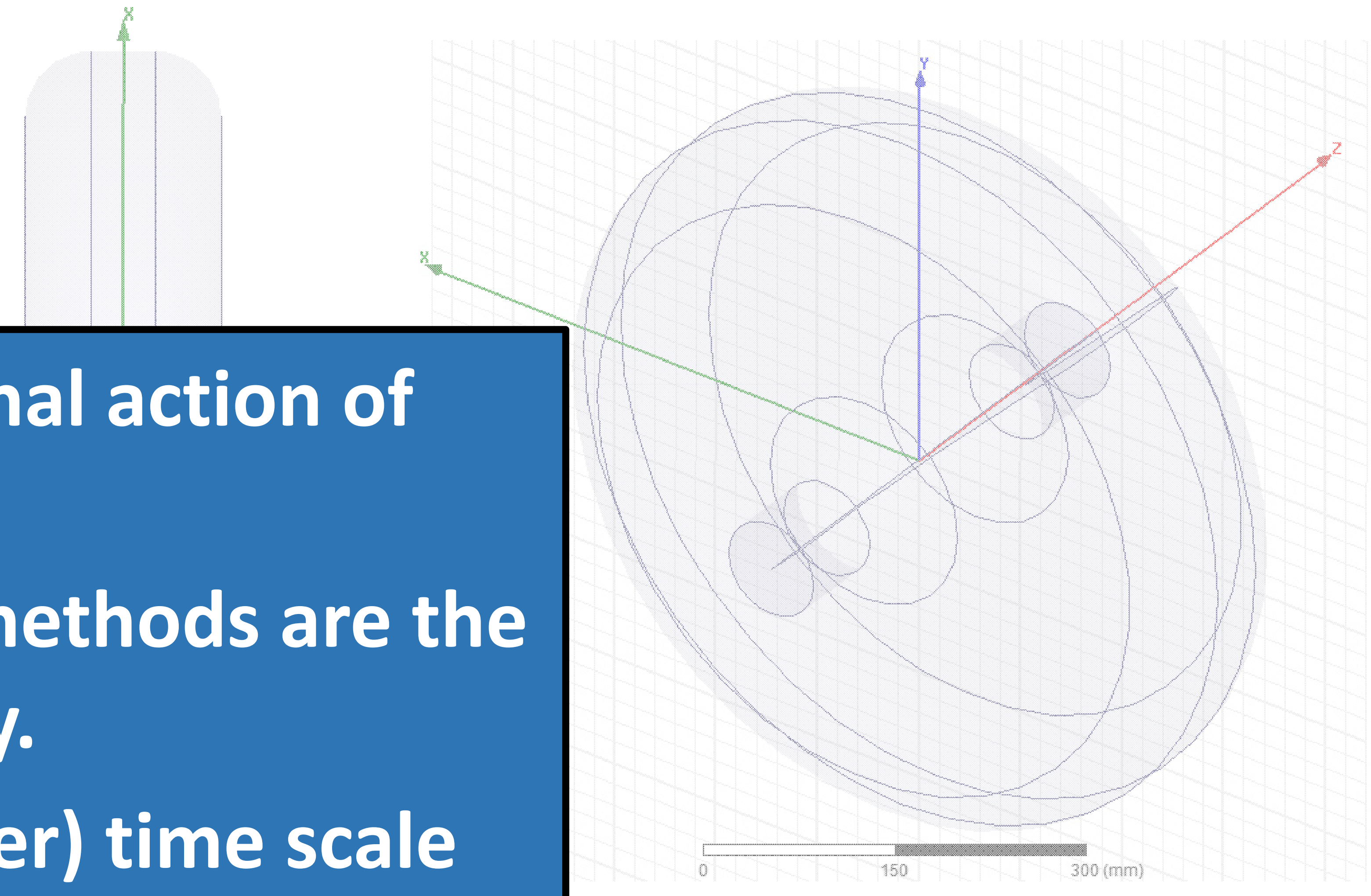

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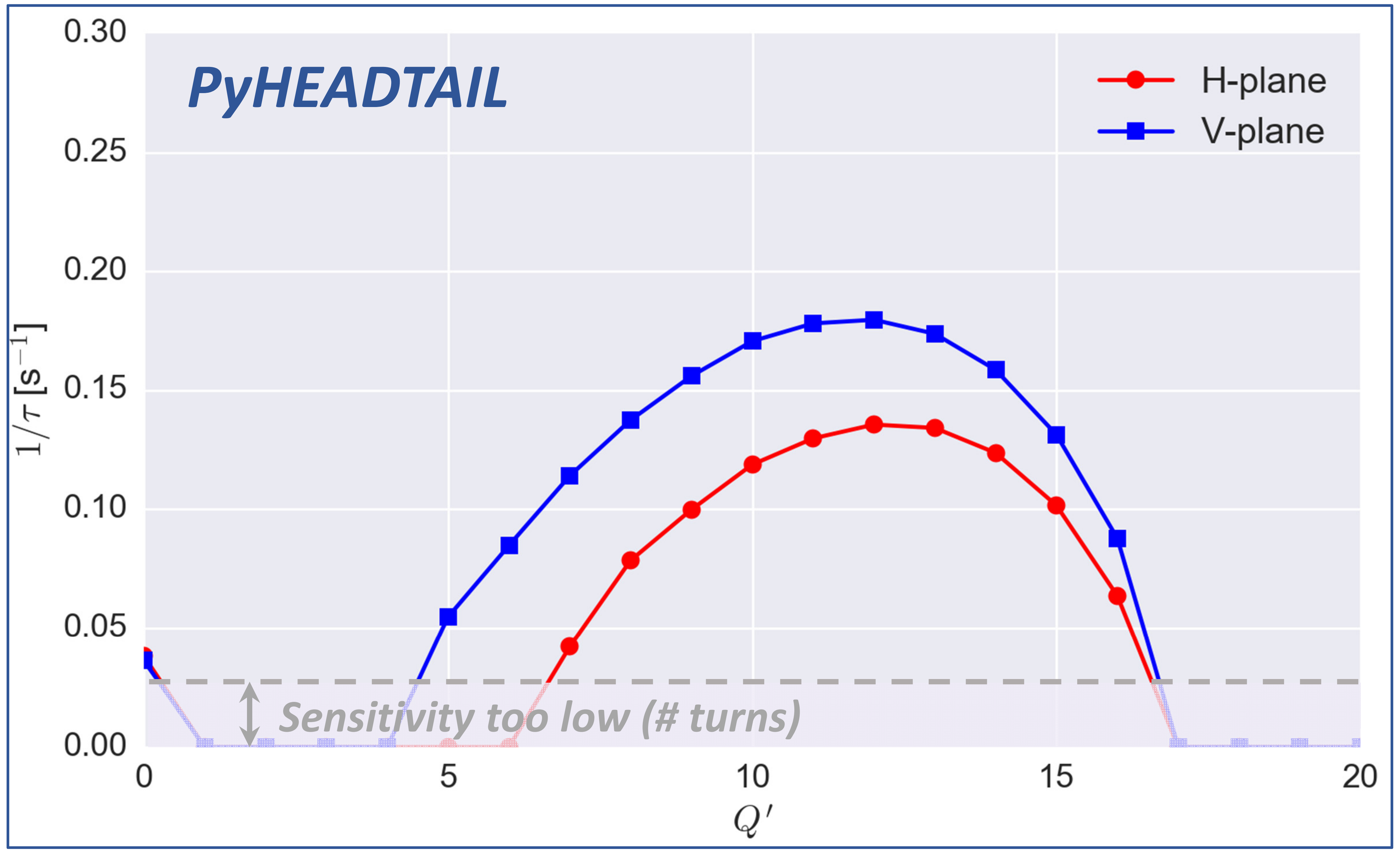
RF quadrupole for HL-LHC
Chromaticity scan

- Single bunch at energy of 7 TeV with nominal beam parameters[7].
- Stabilising systems
  - LHC transverse feedback system, idealised
  - LHC magnetic octupoles aka. **L**andau **O**ctupoles (LO)
  - 800 MHz superconducting RF quadrupole at $\beta_x = \beta_y = 200$ m (conservative)[1-3].
- LHC operation shows that Q' = 10 is a potential working point.



Parameter	HL-LHC 25 ns
Q_x / Q_y	62.31 / 60.32
N_b [p^+]	$2.2\cdot10^{11}$
ε_n [µm]	**2.5**
ε_L [eVs]	**2.5**
V_{RF} [MV]	16

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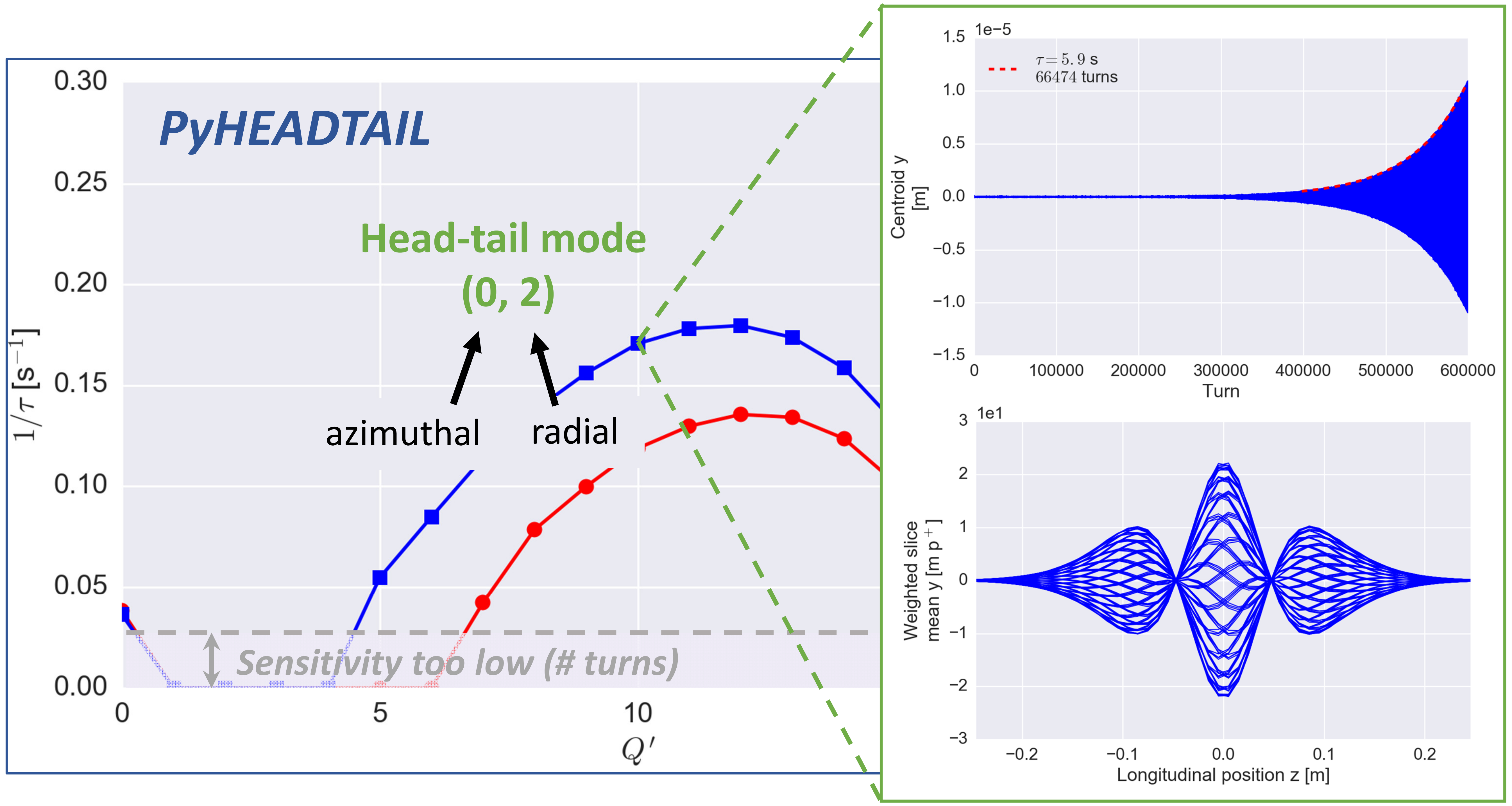
  - 800 MHz superconducting RF quadrupole at $\beta_x = \beta_y = 200$ m (conservative)[1-3].
- LHC operation shows that Q' = 10 is a potential working point.
- Observe head-tail mode (0, 2) in presence of the transverse feedback system.
- Confirmed by measurements in LHC[15].
- Without RF quadrupole, an LO current of $I_f = (170 \pm 10)$ A is required for stabilisation *(taking into account impedance only)*.
- **What is the effect of an RF quadrupole on the required LO current $I_{f,d}$?**



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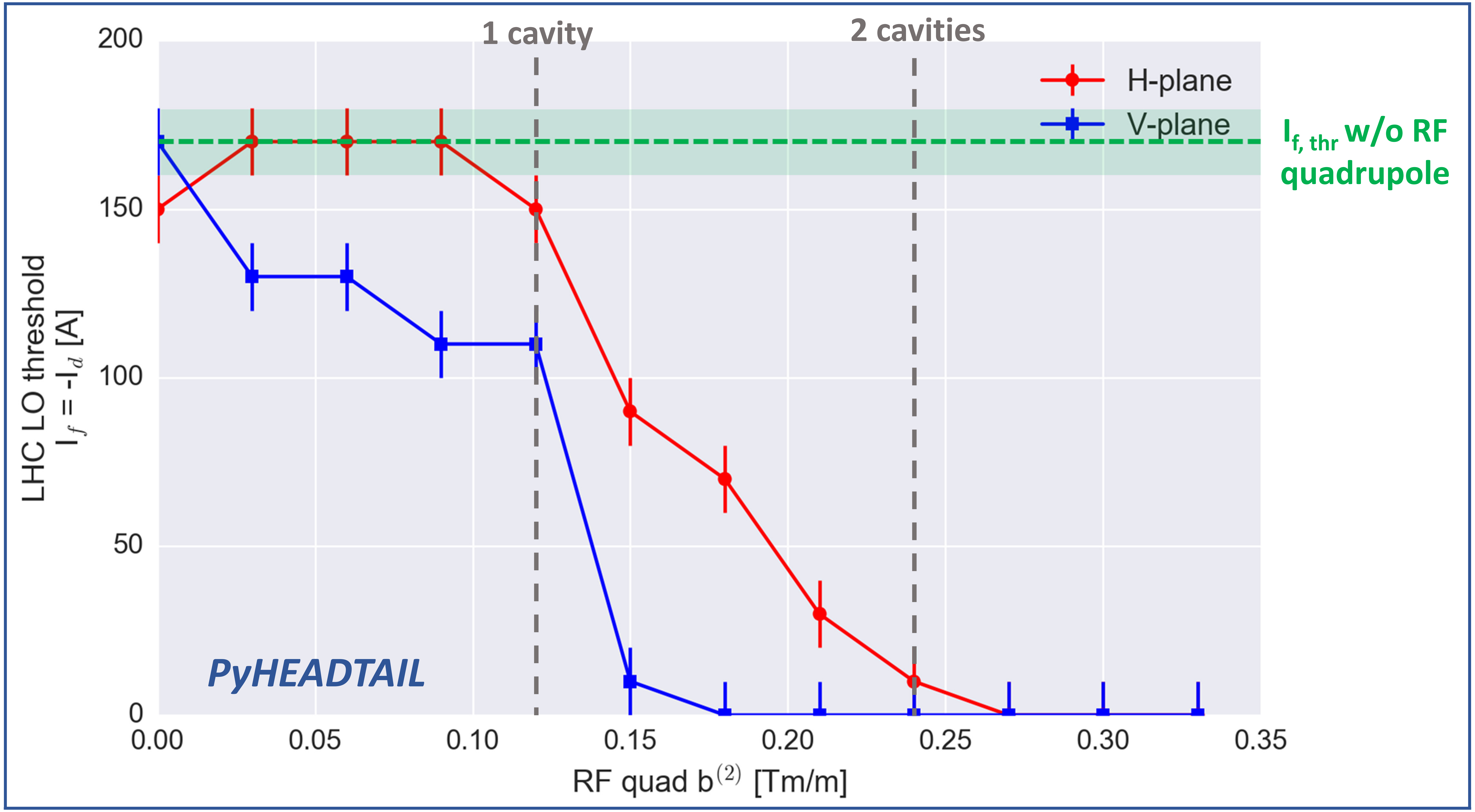
RF quadrupole for HL-LHC

Octupole threshold dependence on RF quadrupole strength



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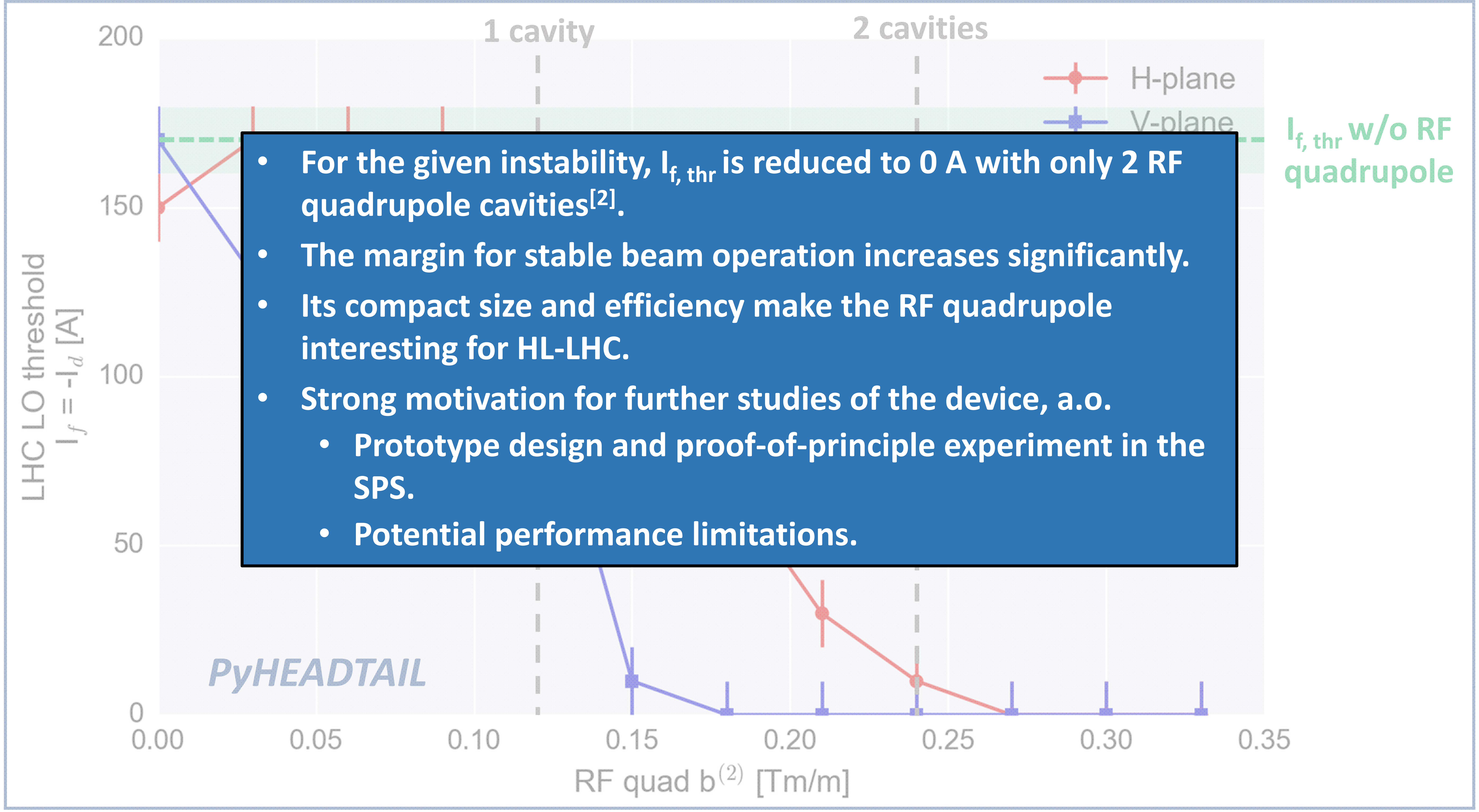

- **For the given instability, $I_{f,\,thr}$ is reduced to 0 A with only 2 RF quadrupole cavities[2].**
- **The margin for stable beam operation increases significantly.**
- **Its compact size and efficiency make the RF quadrupole interesting for HL-LHC.**
- **Strong motivation for further studies of the device, a.o.**
  - **Prototype design and proof-of-principle experiment in the SPS.**
  - **Potential performance limitations.**

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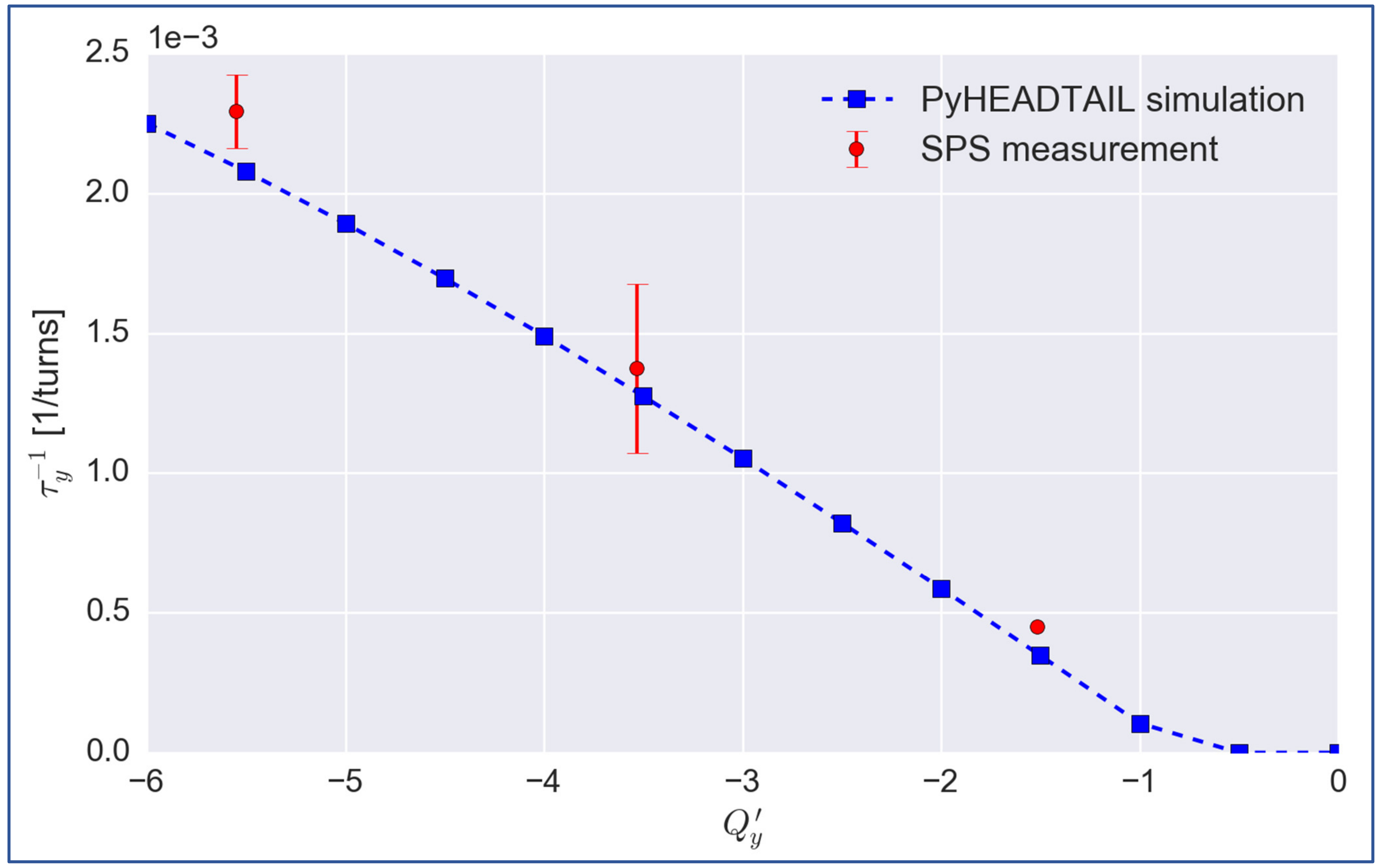
Synchro-betatron resonances

Evaluation of RF quadrupole prototype in SPS I

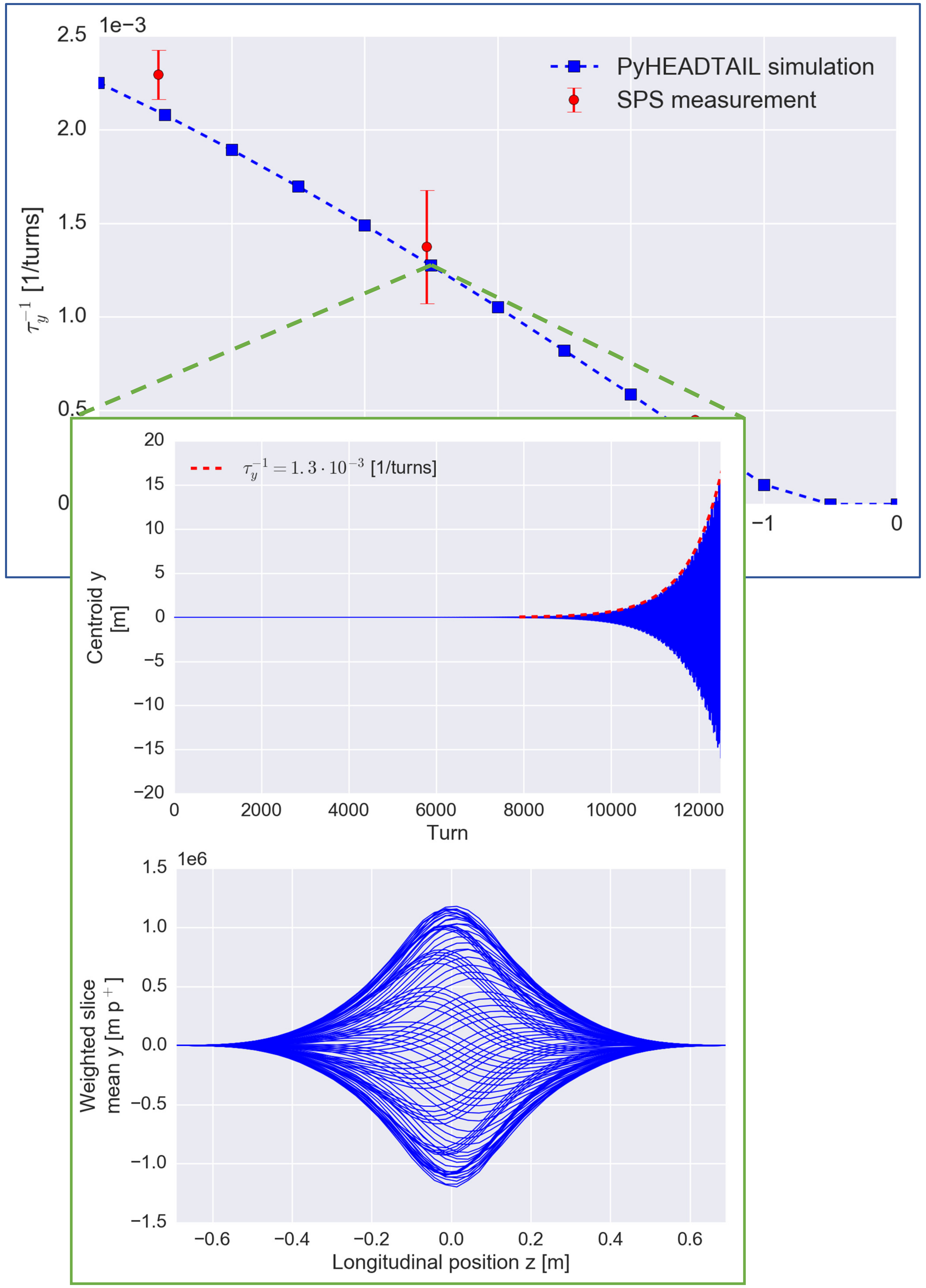
- **For a prototype cavity test in SPS**
  Identify a weak head-tail instability that can be Landau damped.



Synchro-betatron resonances

Evaluation of RF quadrupole prototype in SPS I

- **For a prototype cavity test in SPS**
  Identify a weak head-tail instability that can be Landau damped.
- Focus lies on mode 0 head-tail instability in the vertical plane ($Q'_y < 0$):
  - It is very clear and well-reproducible both in experiment and simulations (reliable impedance model);
  - Good agreement between measurements and PyHEADTAIL simulations;
  - Higher order modes ($Q'_y > 0$) cannot be easily observed experimentally.
- **Can it be stabilised by an RF quadrupole and at what strength $b^{(2)}$?**



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Evaluation of RF quadrupole prototype in SPS II

- Simulations predict that head-tail mode 0 can be stabilised with one to two RF quadrupole cavities.
- By means of an aperture (beam pipe), one can also quantify the particle losses in PyHEADTAIL.
- Allows to identify three regimes.

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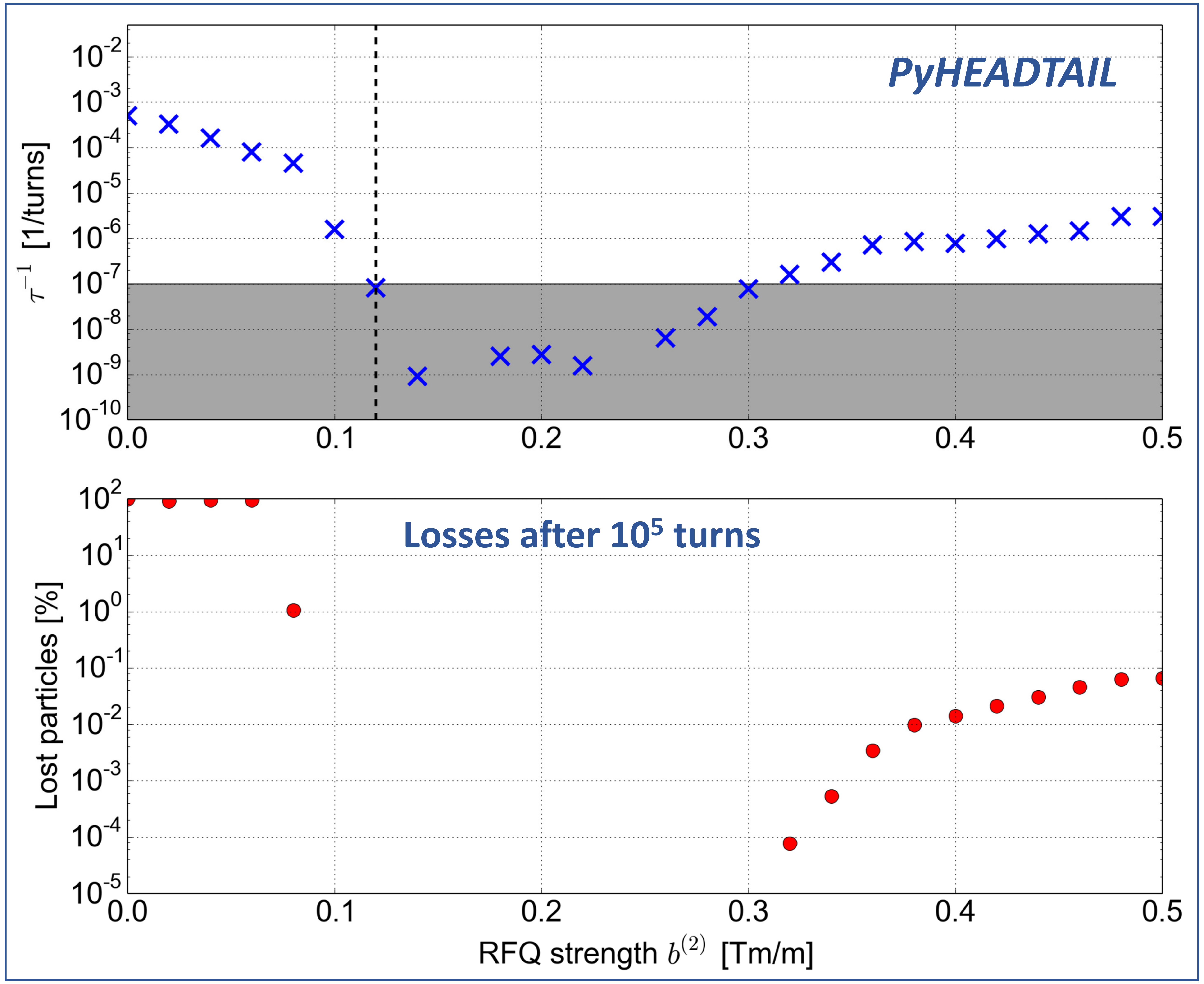

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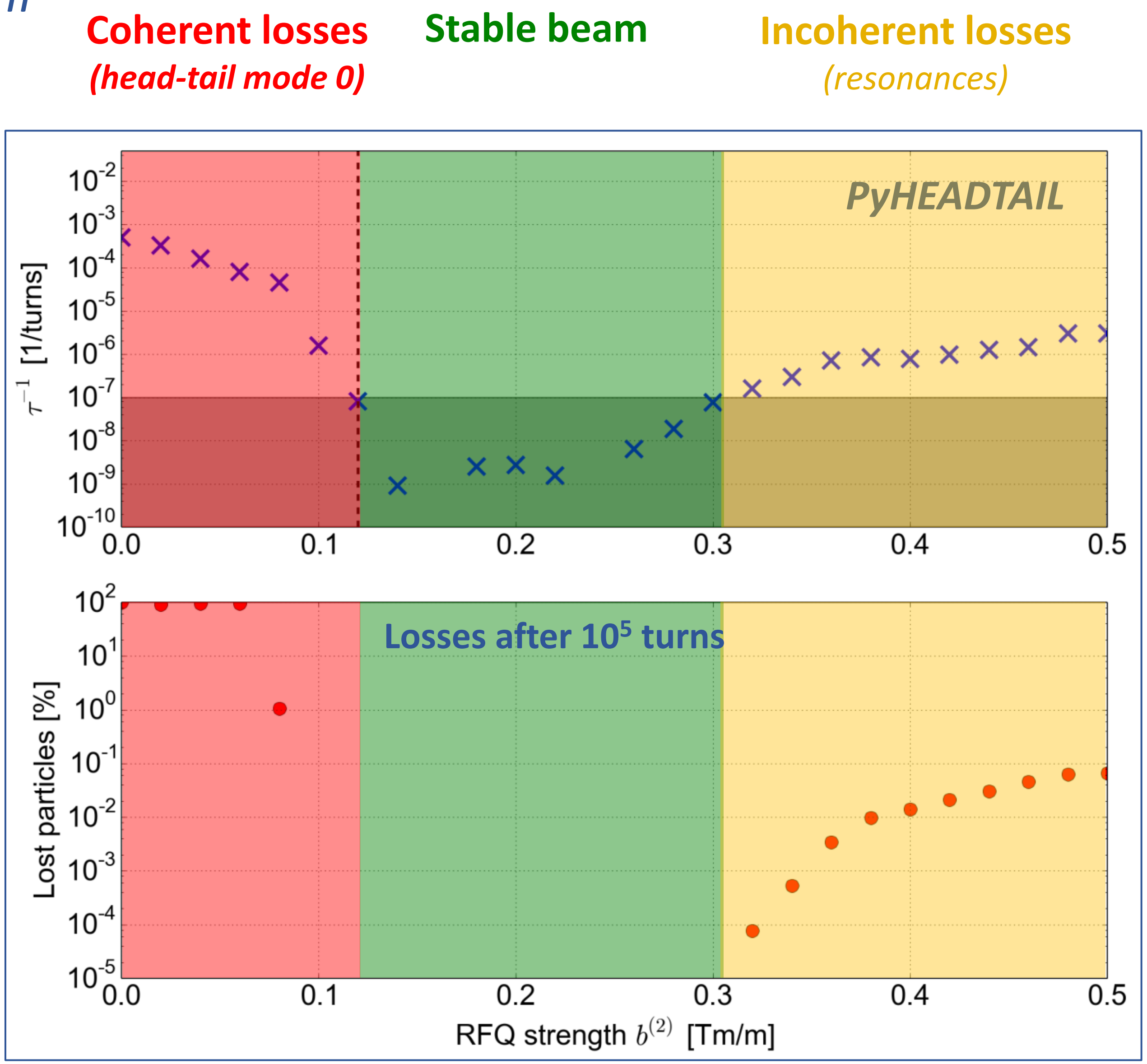

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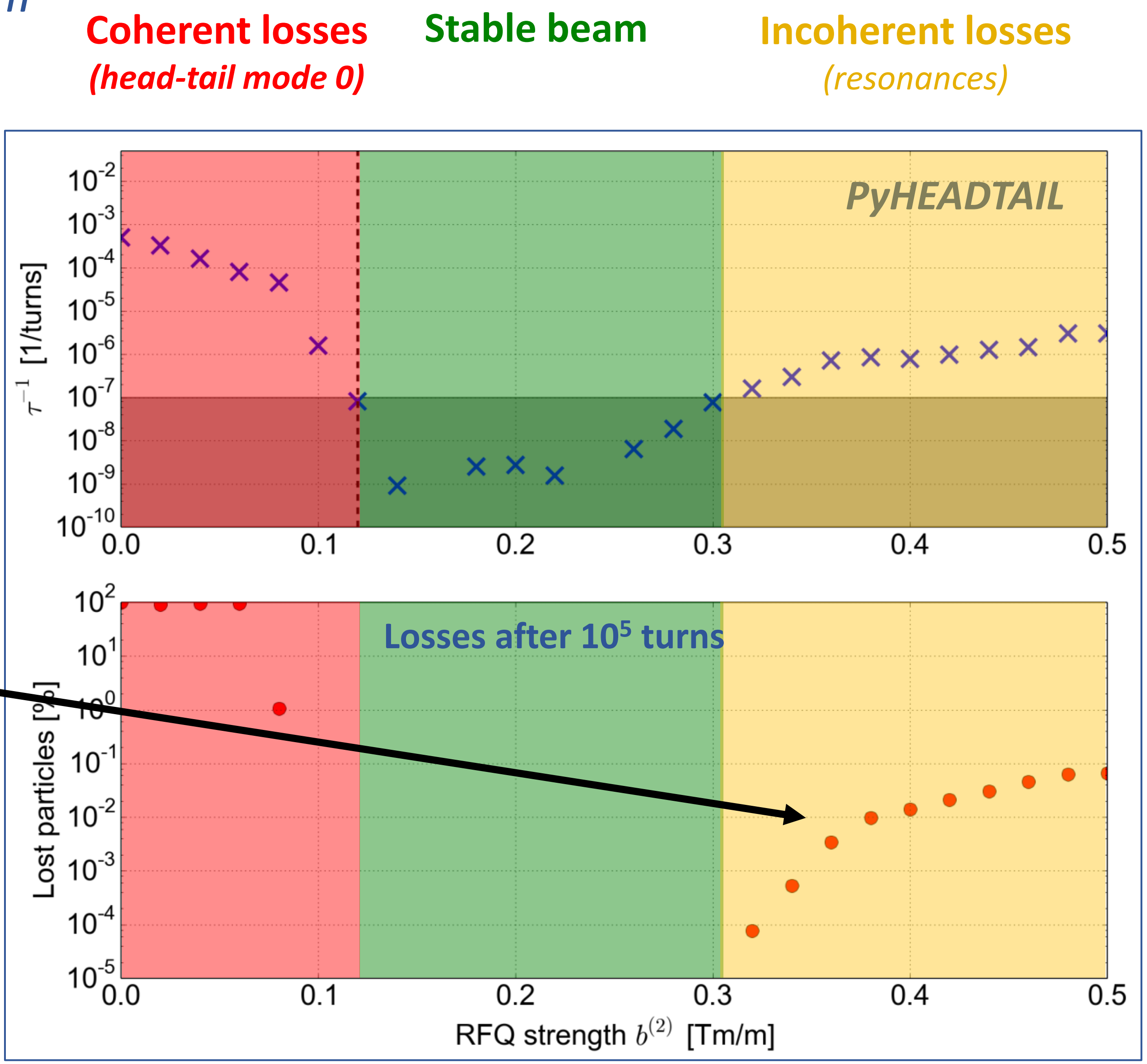
- By means of an aperture (beam pipe), one can also quantify the particle losses in PyHEADTAIL.
- Allows to identify three regimes.
- *Hypothesis:* RF quadrupole excites synchro-betatron resonances at higher strengths[14].
- This is a potential performance limitation which must be studied in detail.



Synchro-betatron resonances

Requirements

- Since an RF quadrupole gives a kick in H (V) as a function of a particle's x (y) and z positions, it can excite **s**ynchro-**b**etatron **r**esonances (SBR)

$$\Delta p_x^i(t) = -q b^{(2)} x_i(t) \cdot \cos\left(\omega \frac{z_i(t)}{\beta c}\right)$$

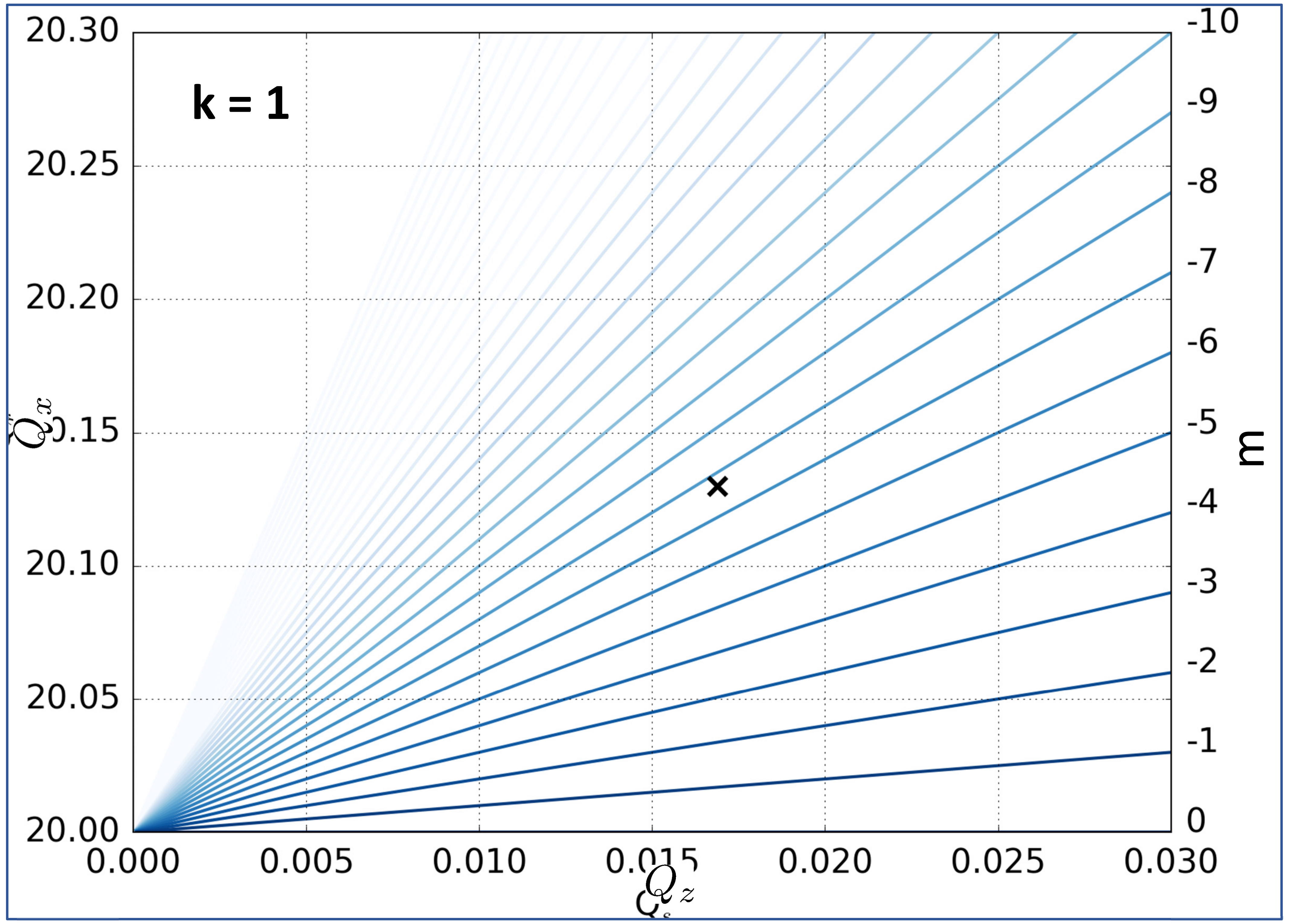
*(note that to first order, the RF quadrupole does not couple x and y)**

- SBR condition is given by the relation[10]

$$k \cdot Q_x + l \cdot Q_y + m \cdot Q_z = n$$
$$\text{with } k,\ l,\ m\ ,\ n \text{ integers}$$

- If we set e.g. l = 0*, the resonance lines in the (Q_z, Q_x) space are given by

$$Q_x = \frac{n - m \cdot Q_z}{k}, \quad k \neq 0$$



Synchro-betatron resonances

Presence of SBR in tracking simulations

Strategy to (dis-)prove presence of SBR

- Observe horizontal action J_x of individual particles *without* and *with* RF quadrupole over several synchrotron periods.
- Separate high transverse action particles (*on-resonance*) from the rest and study if they have distinct betatron and synchrotron tunes.

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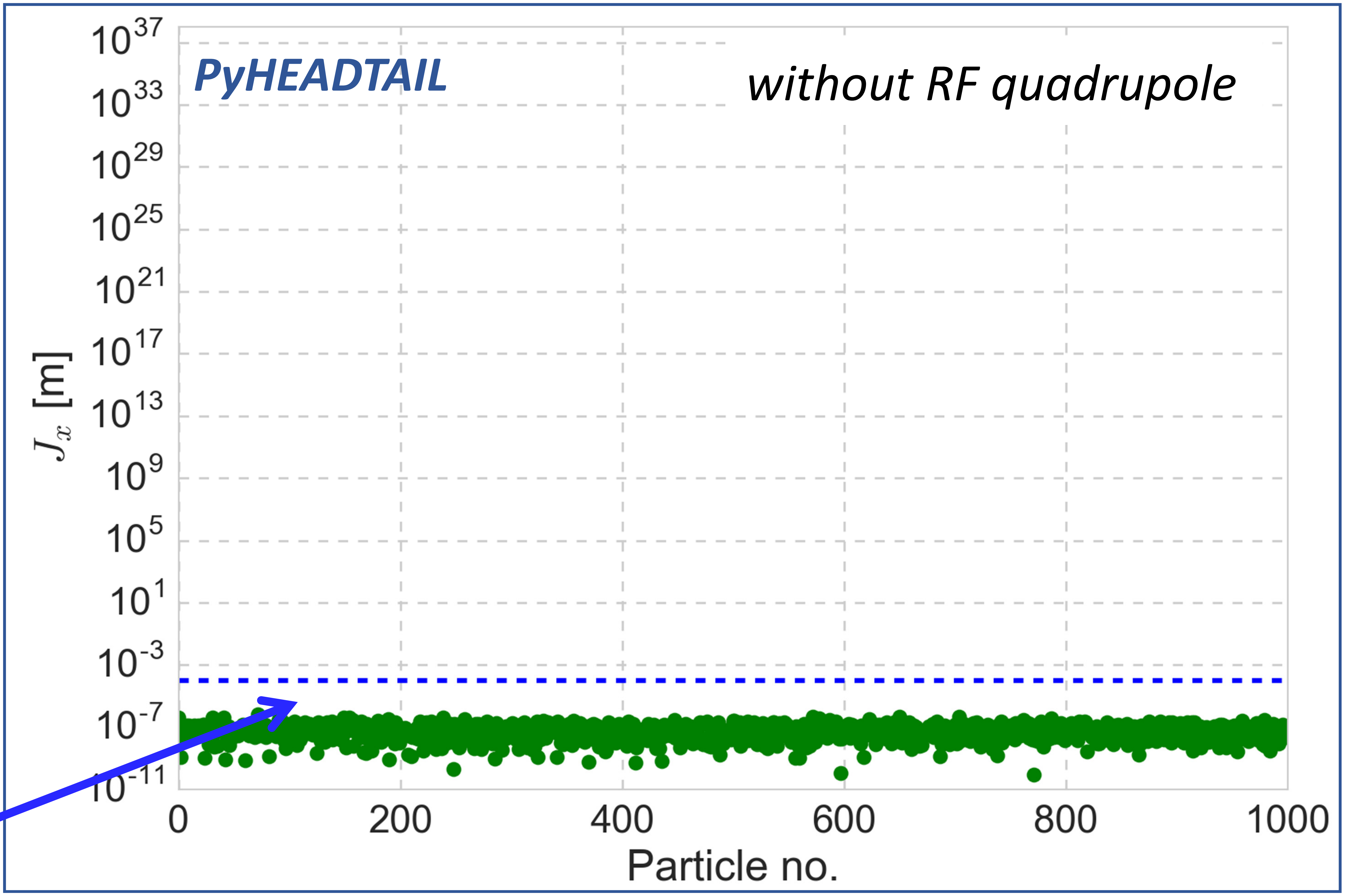

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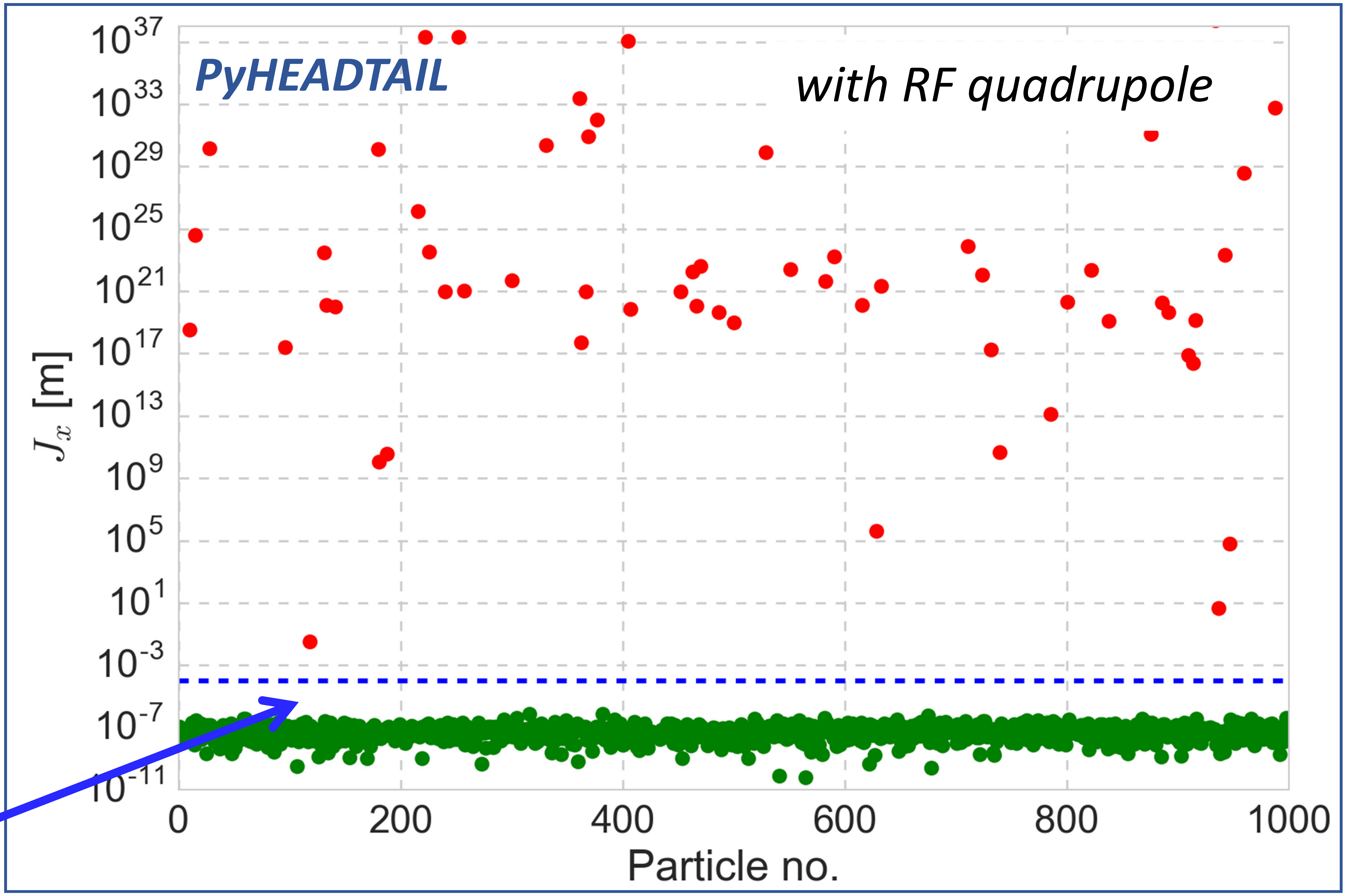

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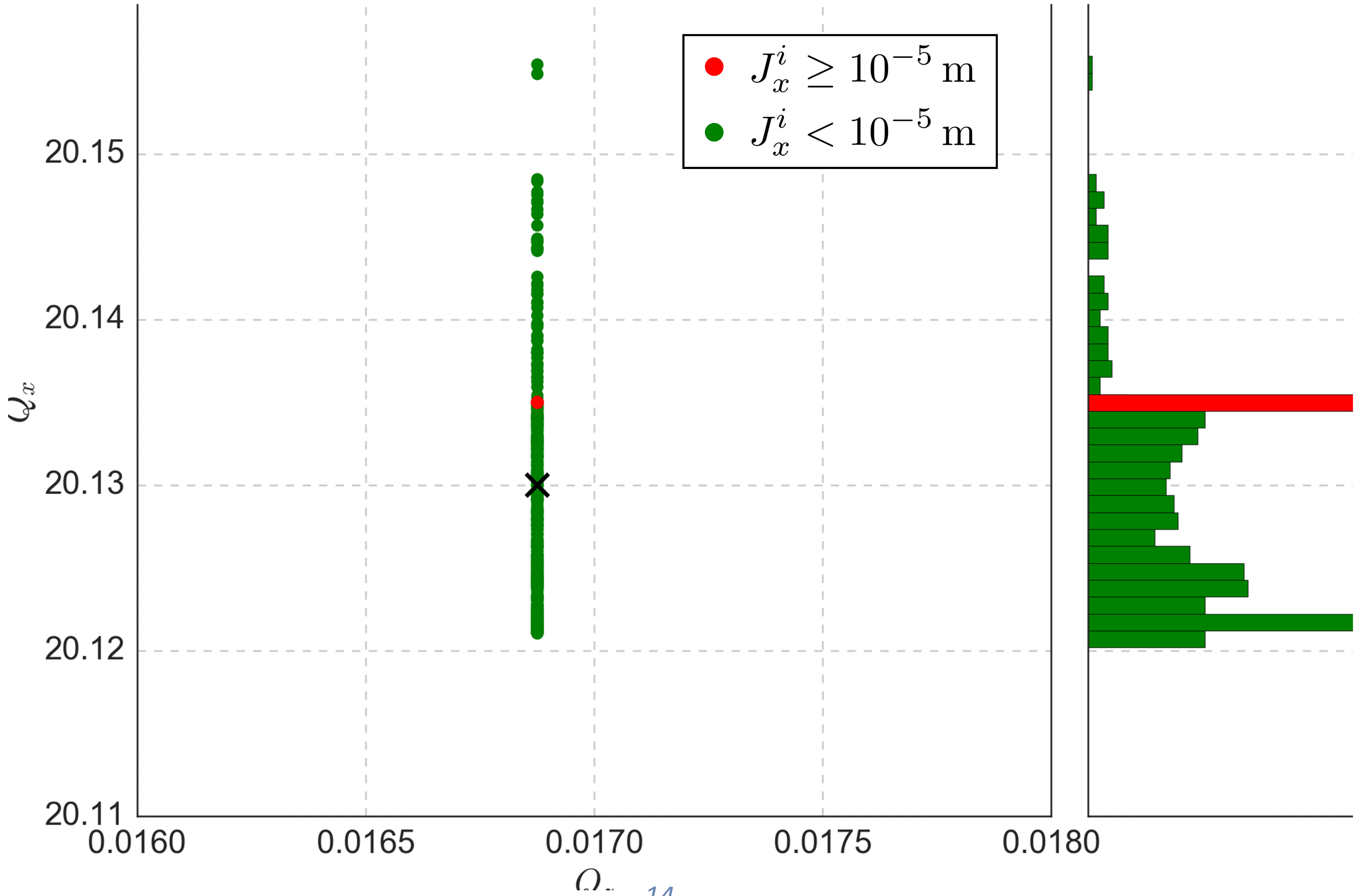

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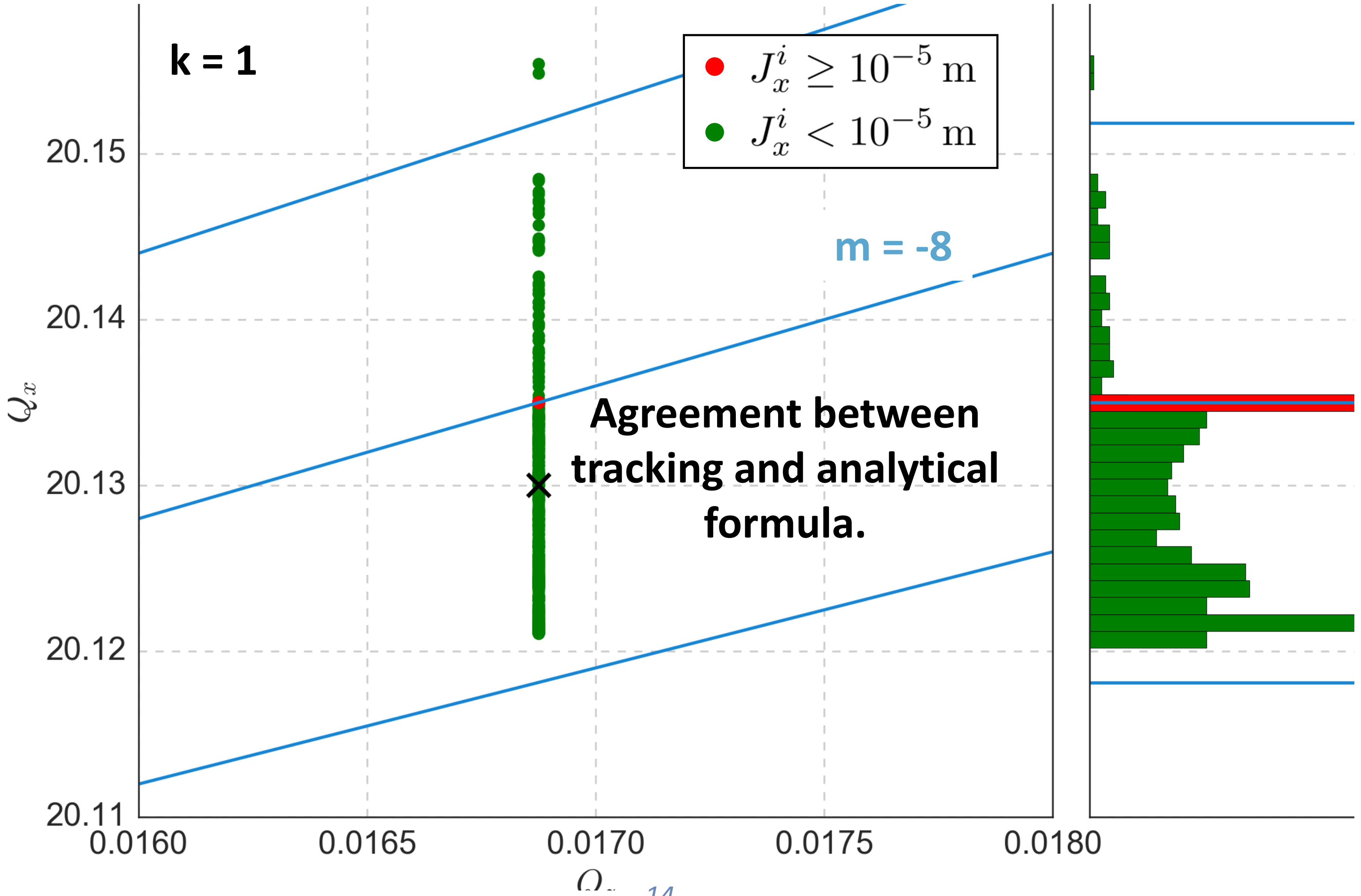

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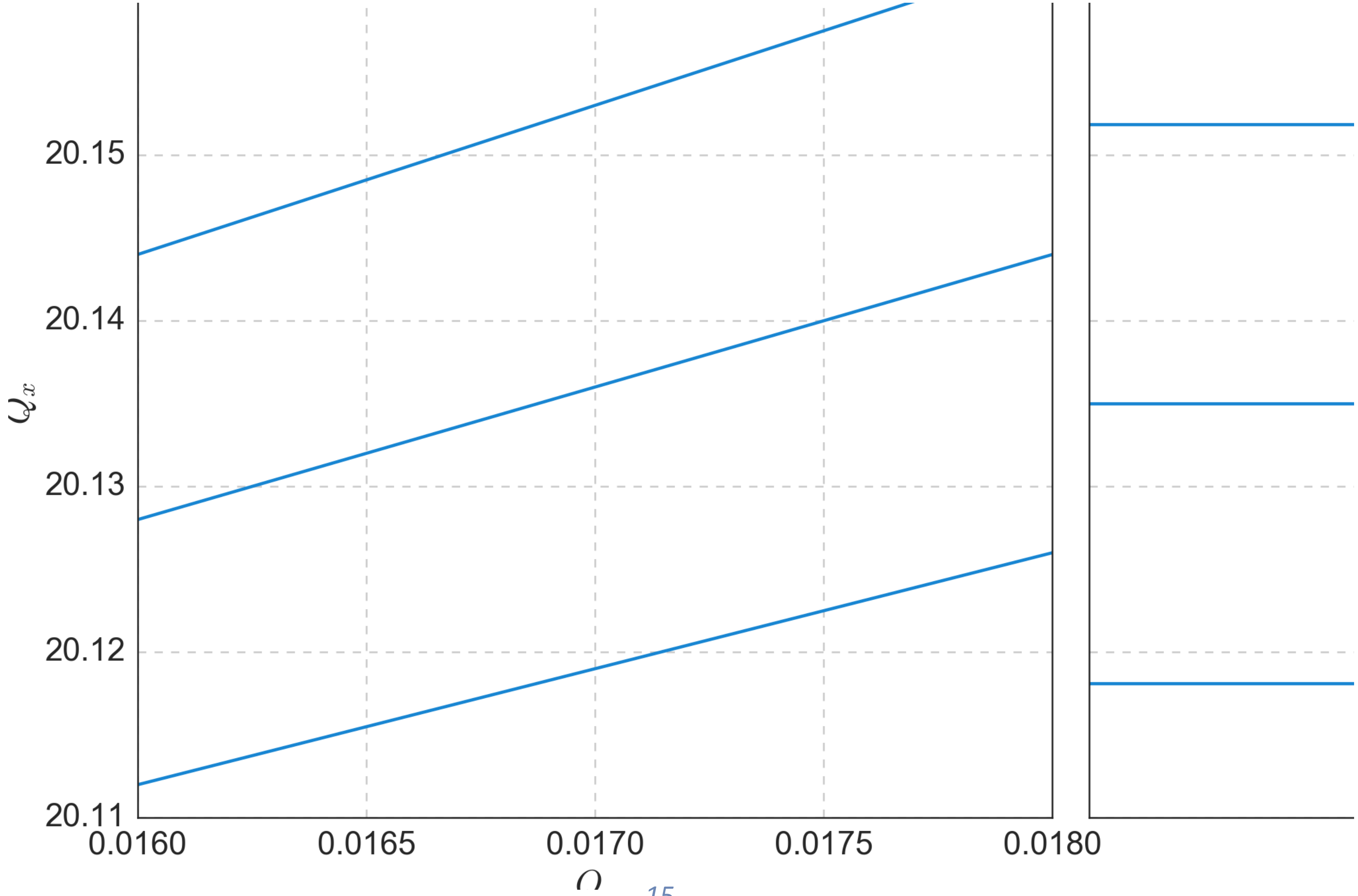
Systematic comparison of tracking and analytical formula



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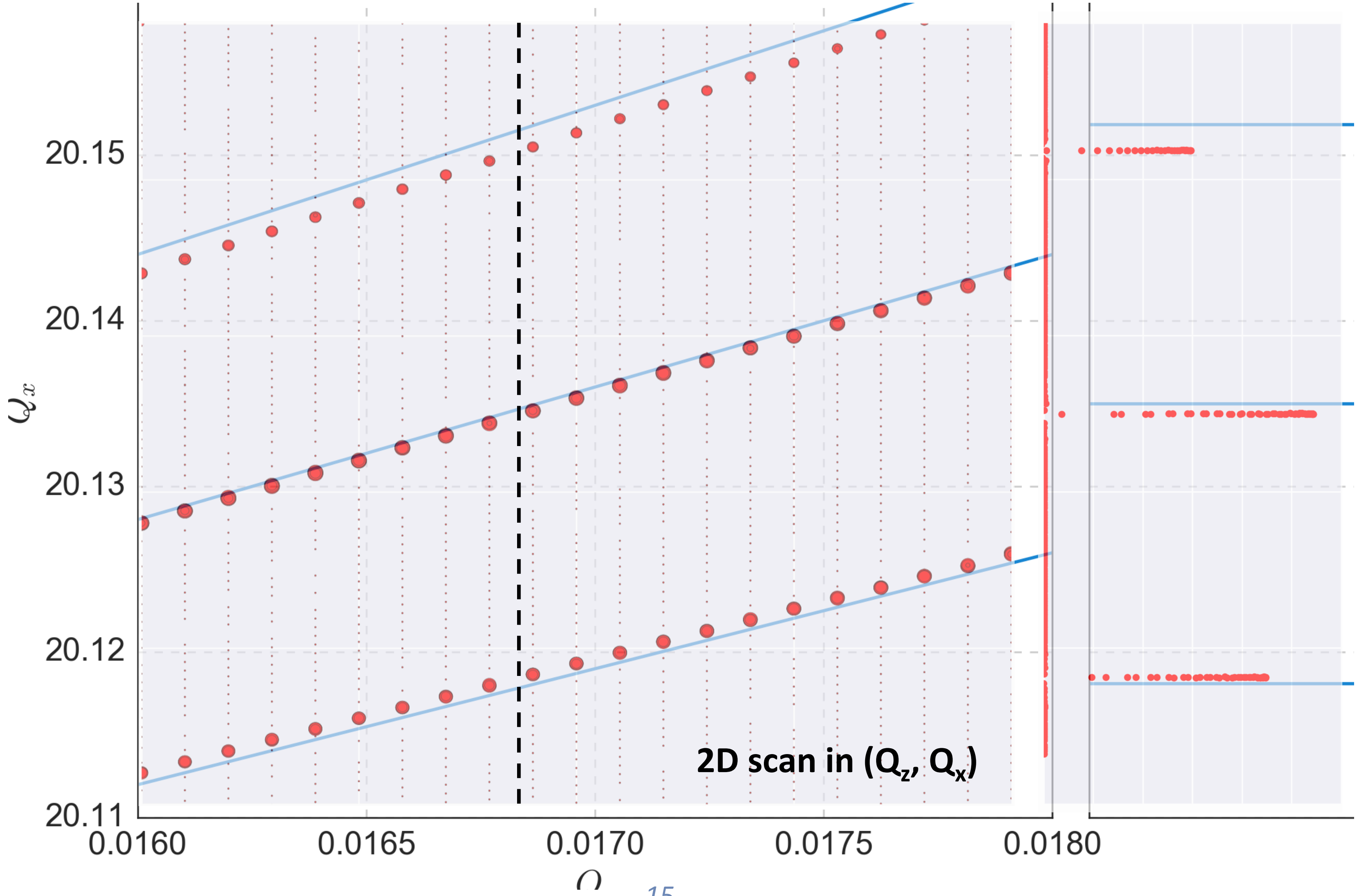

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Stabilisation through Q''

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- Same stabilising mechanism for *Q''* and RF quadrupole, but *Q''* studies can be done without installation of additional equipment.
- Need precise knowledge of the SPS non-linear optics model[11].

Strategy

- Q'' can be controlled by means of magnetic octupoles in regions of high dispersion D_x.
- Powering these elements also introduces detuning with *transverse* amplitude[13]

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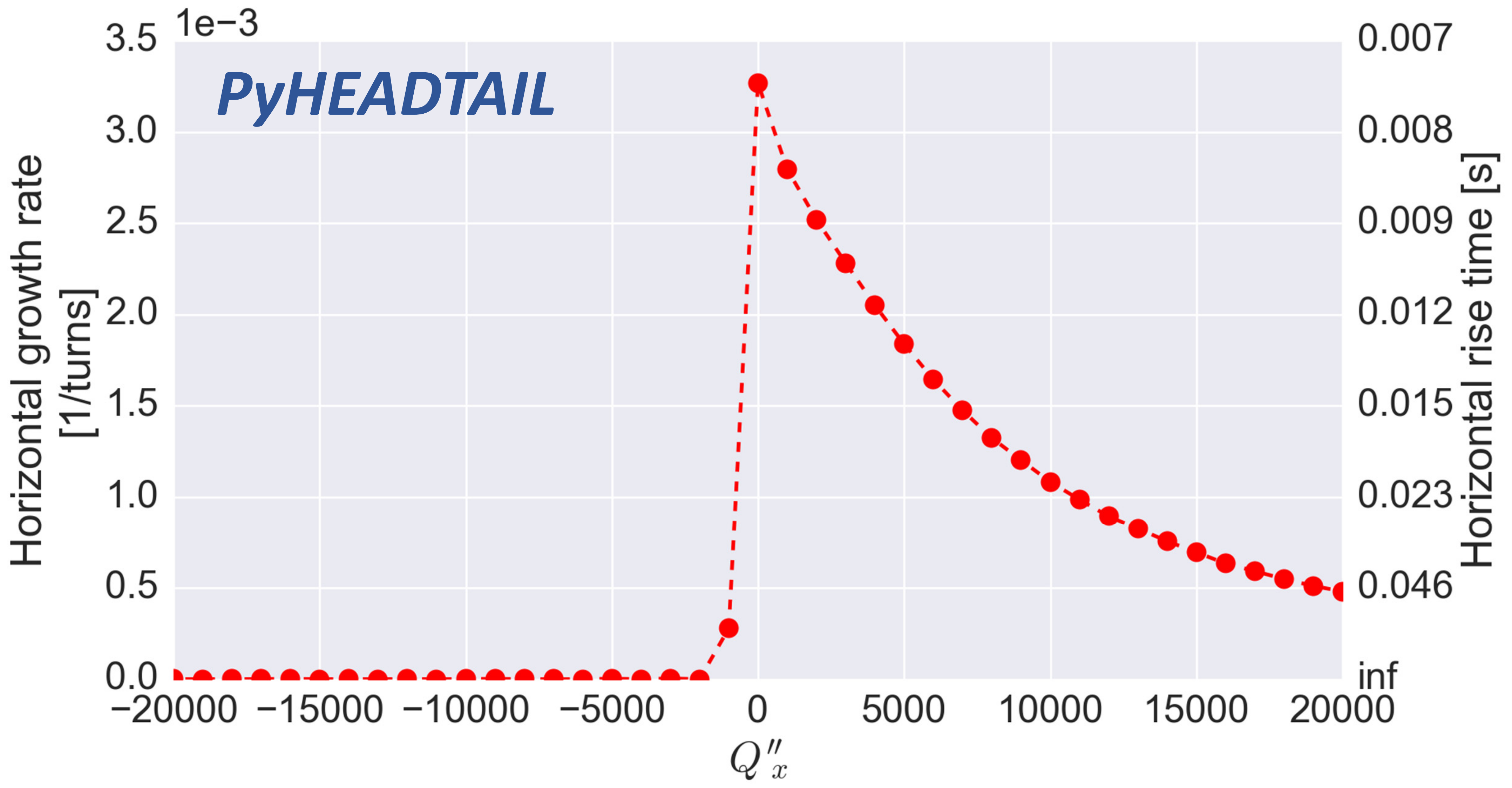
- This must be compensated for to decouple stabilisation through detuning with transverse and longitudinal amplitude respectively.
- Can be achieved by using the magnetic octupoles in low D_x regions as indicated by MAD-X simulations of the SPS optics.

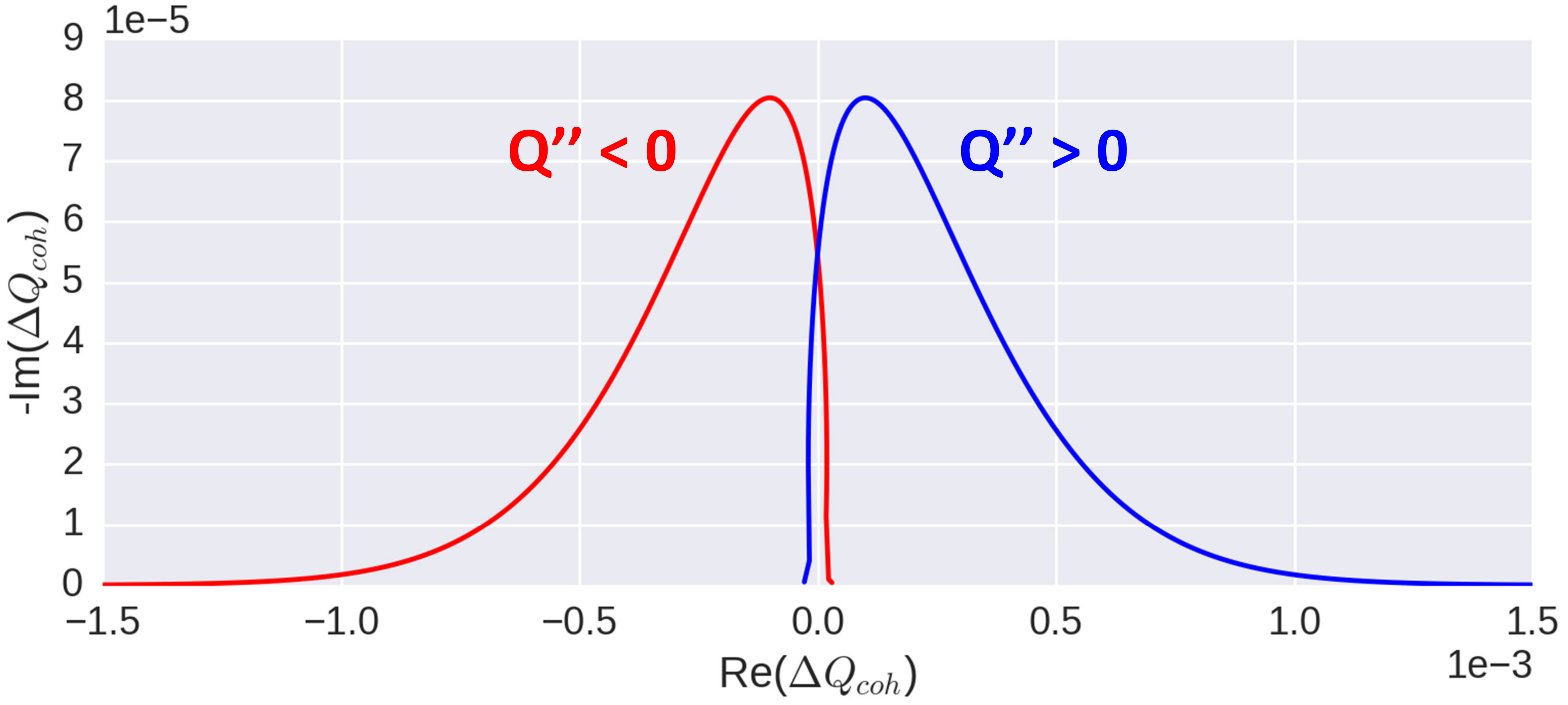
Experimental studies in the SPS

PyHEADTAIL predictions and stability diagram theory



- Horizontal mode 0 head-tail instability in SPS.
- Can be suppressed by non-zero Q''.
- Stabilising behaviour is asymmetric in Q''.
- Q'' < 0 is favourable.

- Solve dispersion integral for transverse Landau damping with longitudinal amplitude[5].
- It can be integrated numerically.
- Stability diagram reflects the asymmetry.



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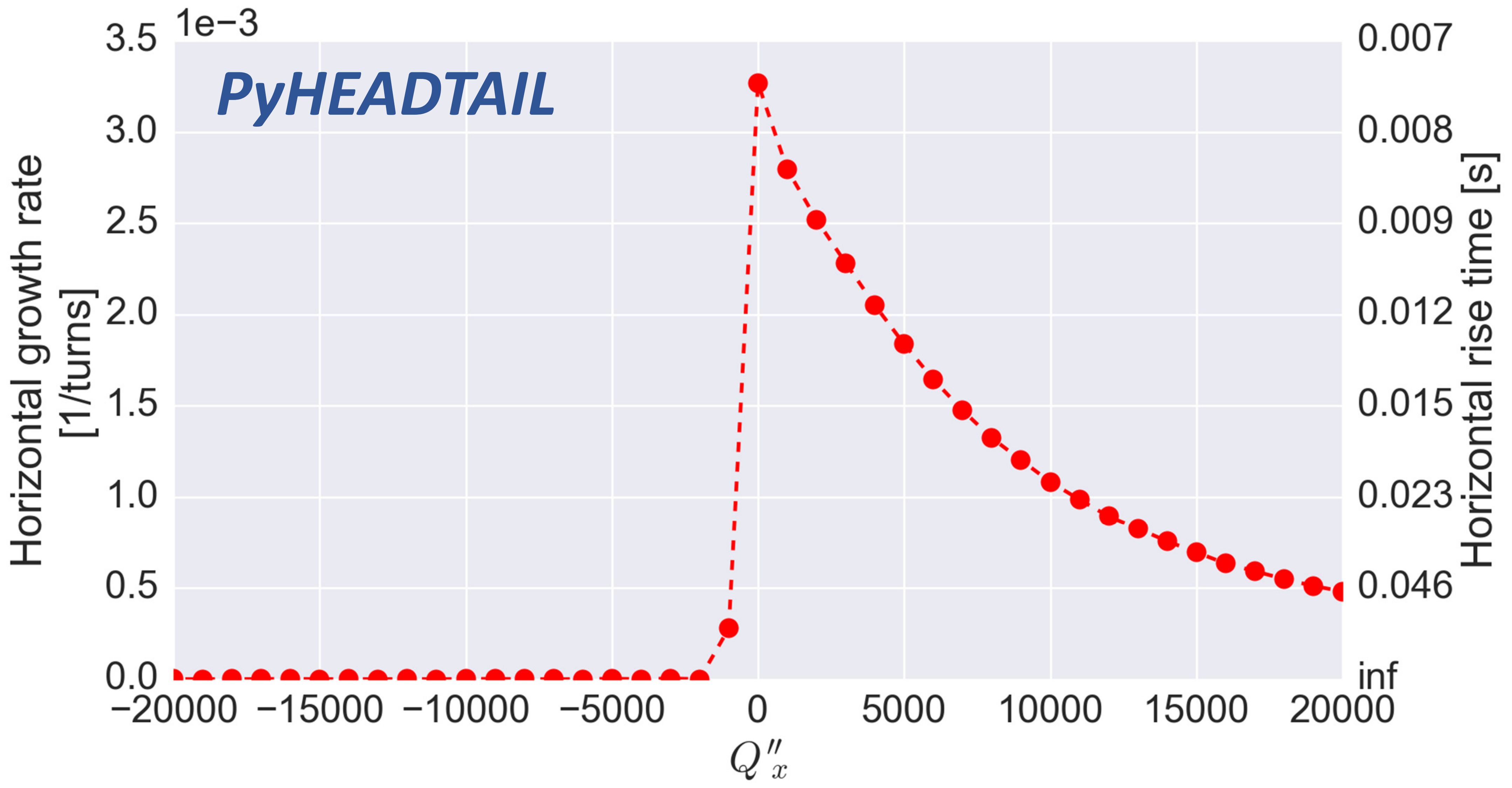

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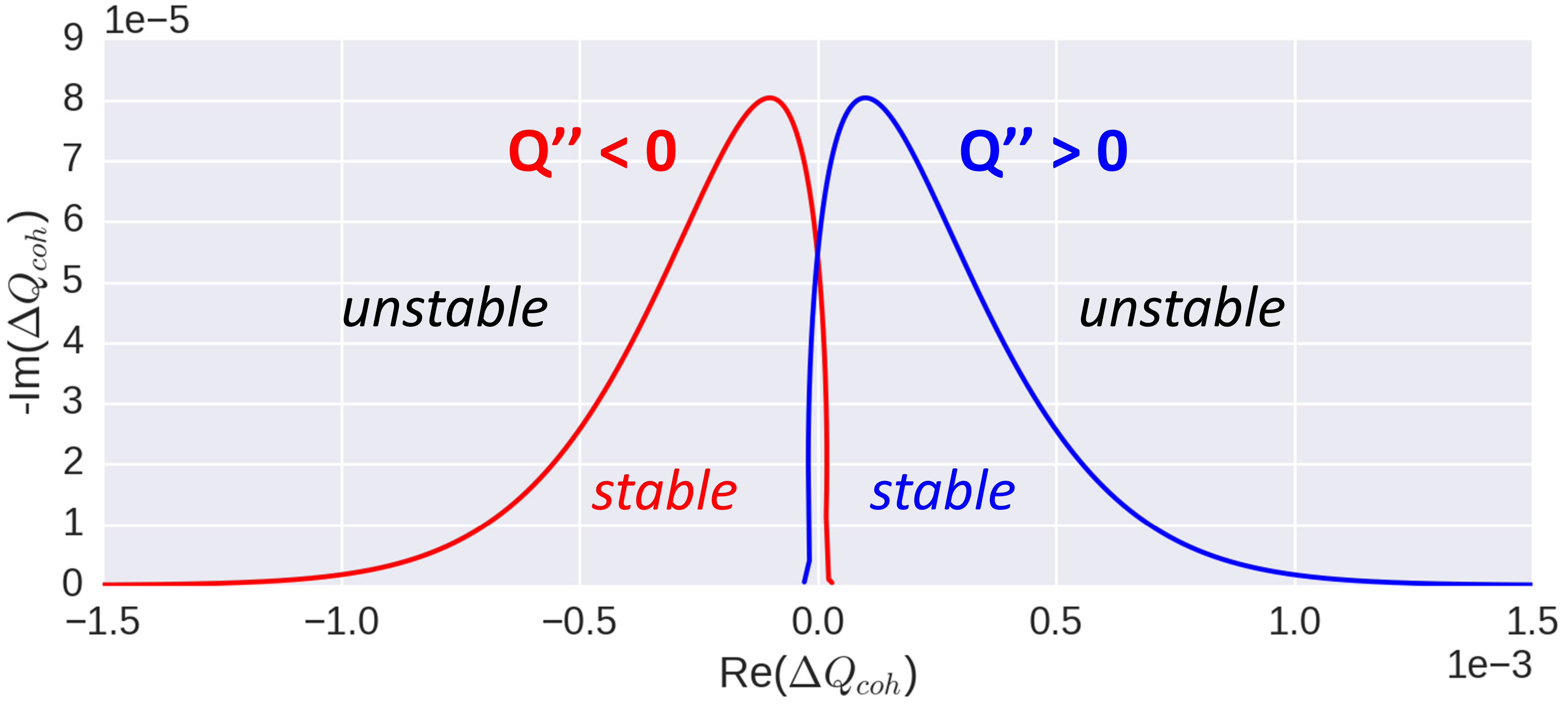

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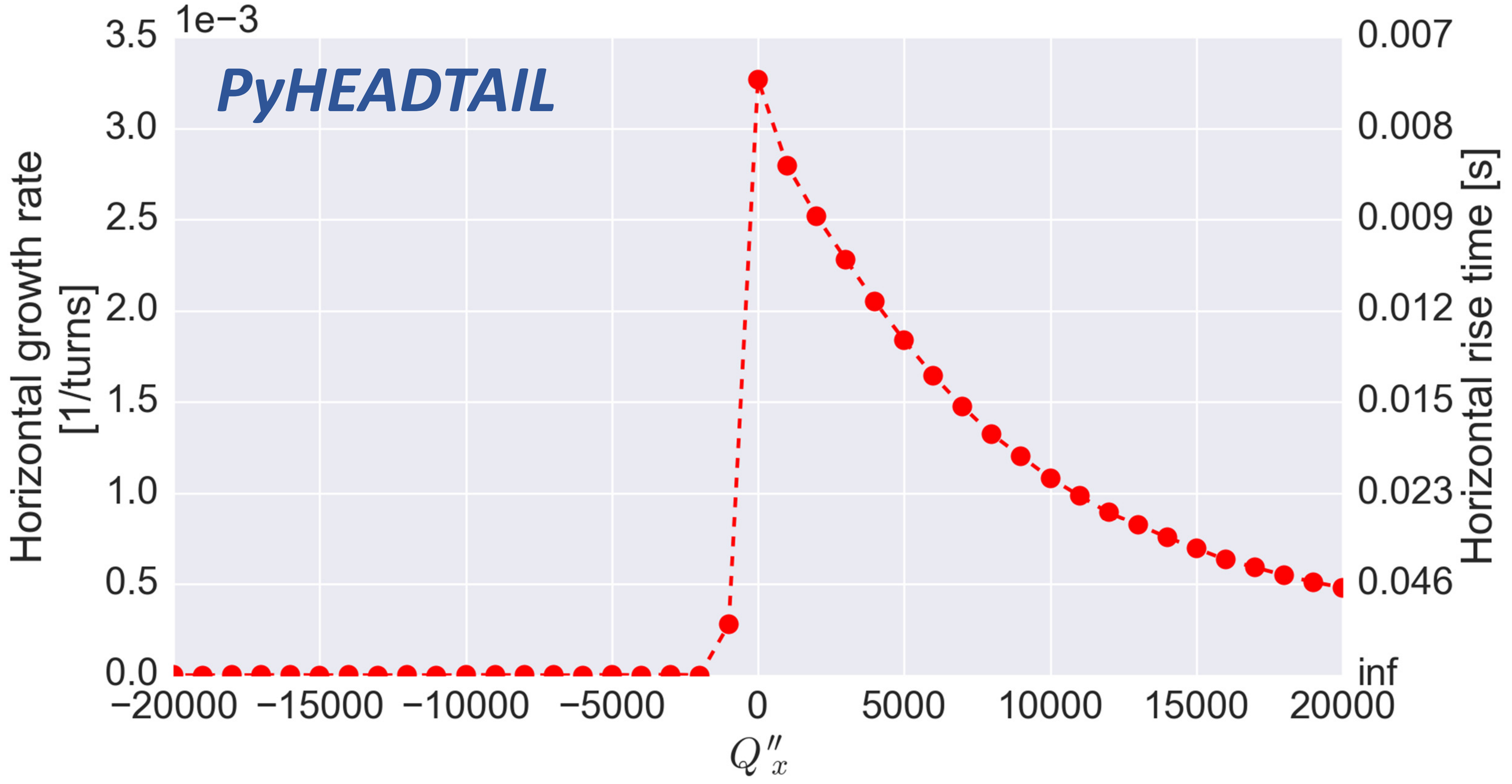

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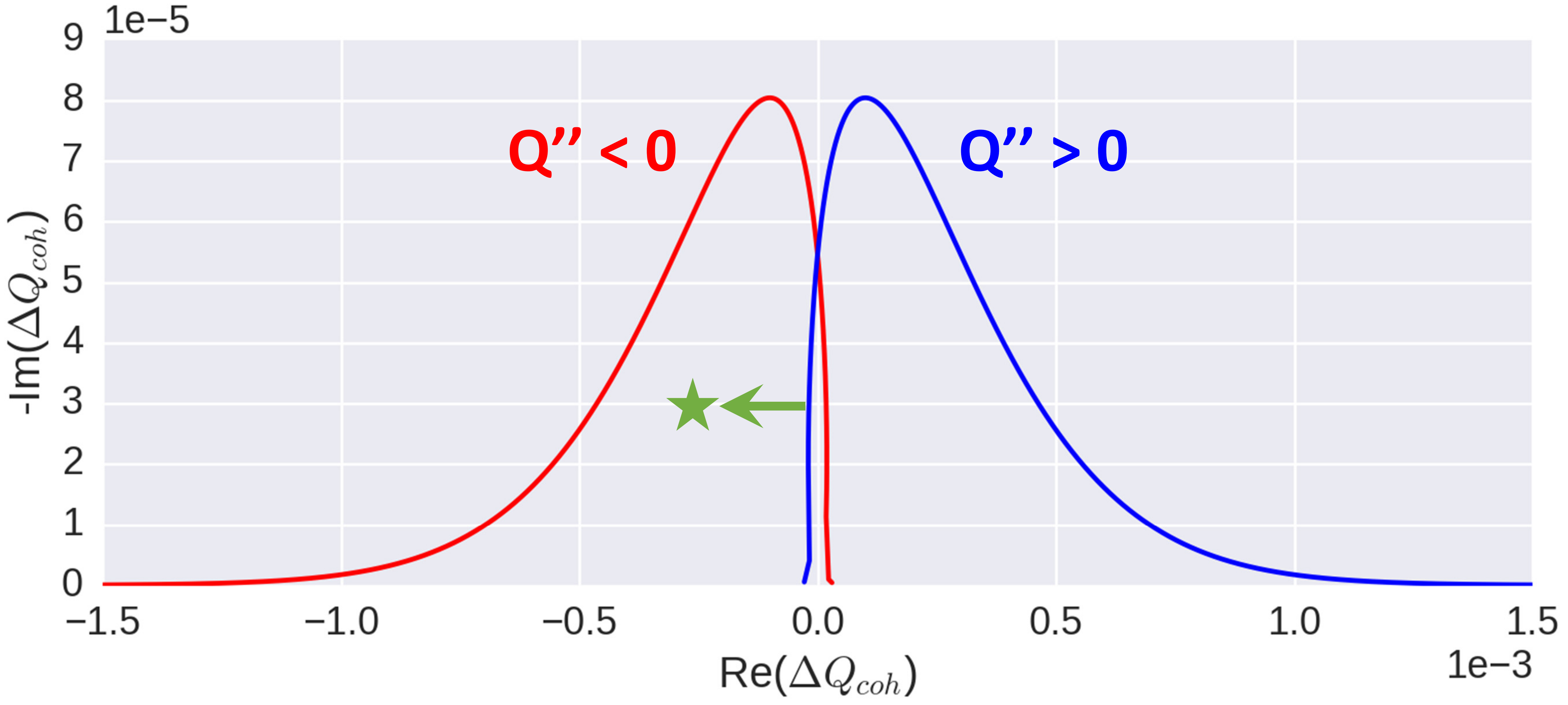
- It can be integrated numerically.
- Stability diagram reflects the asymmetry.
- Dipolar impedance induces *Re(ΔQ$_{coh}$) < 0*.
- In qualitative agreement with tracking.



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- More effective due to the larger spread in longitudinal action compared to transverse.
- Can be introduced e.g. by non-zero Q'' or an RF quadrupole. The underlying stabilising mechanism is the same.
- Numerical simulations with PyHEADTAIL show promising results.
- Potential performance limitations like the excitation of resonances are under study.
- Experimental benchmarks are possible with measurements in SPS using Q''.
- Tracking simulations and stability diagram theory are in qualitative agreement, but quantitative comparison is yet to be done.

Thank you for your attention!

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