Nonlinear Focusing in IOTA for Space-Charge Compensation and Landau Damping

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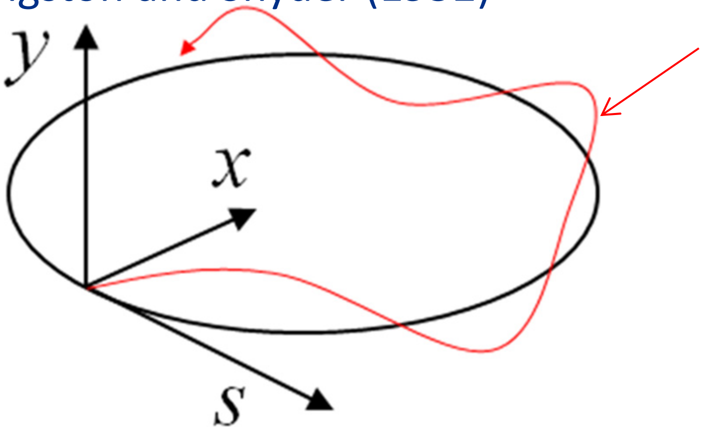
HB 2016

Strong Focusing – Our Standard Approach Since 1952

Christofilos (1949); Courant, Livingston and Snyder (1952)



Particle undergoes betatron oscillations

$$\begin{cases} x'' + K_x(s)x = 0 \\ y'' + K_y(s)y = 0 \end{cases}$$

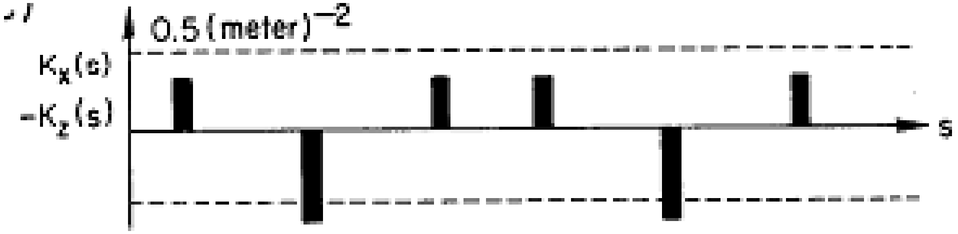
s is "time"

$$K_{x,y}(s+C) = K_{x,y}(s)$$

-- piecewise constant alternating-sign functions

Focusing is provided by *linear* elements, dipoles and quadrupoles

$$H = \frac{1}{2}p_n^2 + \frac{1}{2}x_n^2$$

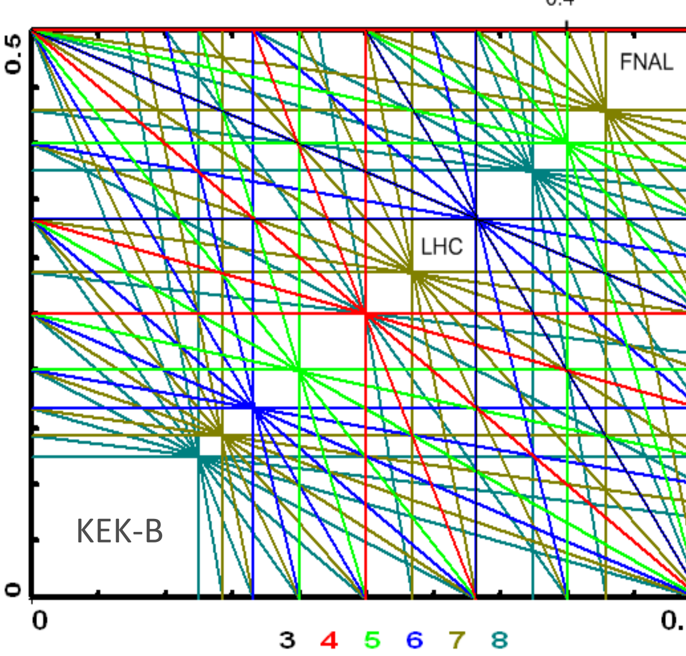


--Magnet lattice and focussing functions in the normal cells of a particular guide field.

Non-linear focusing

- It became obvious very early on (~1960), that the use of nonlinear focusing elements in rings is necessary and some nonlinearities are **unavoidable** (magnet aberrations, space-charge forces, beam-beam forces)
  - Sexupoles appeared in 1960s for chromaticity corrections
  - Octupoles were installed in CERN PS in 1959 but not used until 1968. For example, the LHC has ~350 octupoles for Landau damping.
- It was also understood at the same time, that nonlinear focusing elements have both beneficial and detrimental effects, such as:
  - They drive nonlinear resonances (resulting in particle losses) and decrease the dynamic aperture (also particle losses).

KAM theory



- Developed by Kolmogorov, Arnold, Moser (1954-63).
- Explains why we can operate accelerators away from resonances.
- The KAM theory states that if the system is subjected to a weak nonlinear perturbation, some of periodic orbits survive, while others are destroyed. The ones that survive are those that have “sufficiently irrational” frequencies (this is known as the non-resonance condition).
- Does not explain how to get rid of resonances
  - Obviously, for accelerators, making ALL nonlinearities to be ZERO would reduce (or eliminate) resonances
  - However, nonlinearities are necessary and unavoidable.

Nonlinear Integrable Systems

- **Are there "magic" nonlinearities with zero resonance strength?**
- **The answer is – *yes* (we call them "*integrable*")**
- Integrable means "having sufficient number of conserved quantities (integrals of motion)". Need two integrals of motion for transverse focusing (a 2-d system)
  - Strong focusing is a linear integrable system; two integrals of motion are the Courant-Snyder invariants
- There many integrable dynamical systems, but we know only a handful suitable for accelerators
- What we are looking for is a non-linear equivalent to Courant-Snyder invariants, for example

$$H = \frac{1}{2}(p_x^2 + p_y^2) + \frac{1}{2}(x^2 + y^2) + \frac{\alpha}{4}\left(x^4 + y^4\right)$$

Specifics of accelerator focusing

- The transverse focusing system is 2.5D (i.e. time-dependent)
    - In a linear system (strong focusing), the time dependence can be transformed away by introducing a new “time” variable (the betatron phase advance). Thus, we have the Courant-Snyder invariant.
- The focusing elements we use in accelerator must satisfy:
    - The Laplace equation (for static fields in vacuum)
    - The Poisson equation (for devices based on charge distributions, such as electron lenses or beam-beam)

History of searches

Searches for nonlinear stable accelerator focusing systems:

- Orlov (1963) – first ideas, no practical solution
- McMillan (1967) – 1D solution
- ✓ Perevedentsev, Danilov (1990) – generalization of McMillan case to 2D, round colliding beams. **Require non-Laplacian potentials to realize**
  - Round colliding beams possess 1 invariant – VEPP-2000 at BINP (Novosibirsk, Russia) commissioned in 2006. Record-high beam-beam tune shift ~0.25 attained in 2013
- Danilov, Shiltsev (1998) – Non-linear electron lenses suggested, FNAL-FN-0671
- Chow, Cary (1994)
- ✓ Nonlinear Integrable Optics: Danilov and SN solution for nonlinear lattice with 2 and 1 invariants of motion that **can be implemented with a Laplacian potential**, i.e. with special magnets – *Phys. Rev. ST Accel. Beams 13, 084002 (2010)*

Why are we doing it?

- Broad scientific interest in integrable dynamical systems.
- Better understanding of beam dynamics.
- Potential practical implementations for:
  - Enhanced Landau damping in rings without degrading of the dynamic aperture;
  - Space-charge compensation; potential reduction of a space-charge driven beam halo;
  - Mitigation of beam-beam effects;

Example 1

- Conceptually, we (at Fermilab) know now how to make a focusing system (with quadrupoles and thin octupoles), which results in the following 2D integrable nonlinear Hamiltonian

$$H=\frac{1}{2}(p_{nx}^2+p_{ny}^2)+\frac{1}{2}(x_n^2+y_n^2)+\frac{\alpha}{4}\left(x_n^4+y_n^4\right)$$

OR

$$H=\frac{1}{2}(p_{nx}^2+p_{ny}^2)+\frac{1}{2}(x_n^2+y_n^2)+\frac{\alpha}{4}\left(x_n^2+y_n^2\right)^2$$

In normalized variables

$$x_n=\frac{x}{\sqrt{\beta(s)}},$$

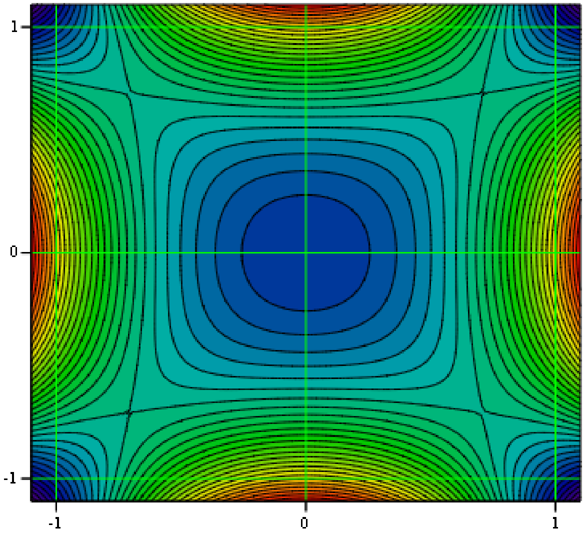
$$p_n=p\sqrt{\beta(s)}-\frac{\beta'(s)x}{2\sqrt{\beta(s)}},$$

- This concept is impractical but very important as it may serve as a model for modeling studies.

Example 2

- A nonlinear partially-integrable focusing system with one integral of motion. Can be implemented in practice (with octupoles). This is one of the systems we plan for IOTA
- A Henon-Heiles type system

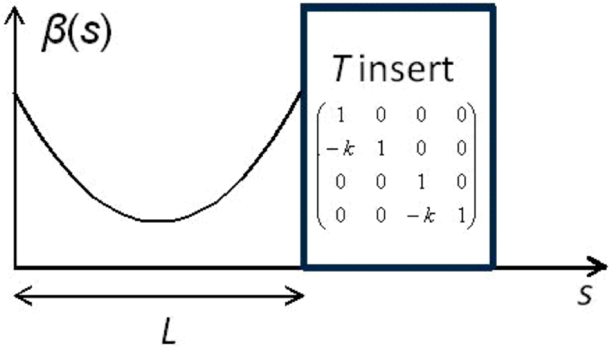
$$H = \frac{1}{2}(p_{nx}^2 + p_{ny}^2) + \frac{1}{2}(x_n^2 + y_n^2) + \frac{\alpha}{4}\left(x_n^4 + y_n^4 - 6x_n^2 y_n^2\right)$$



B

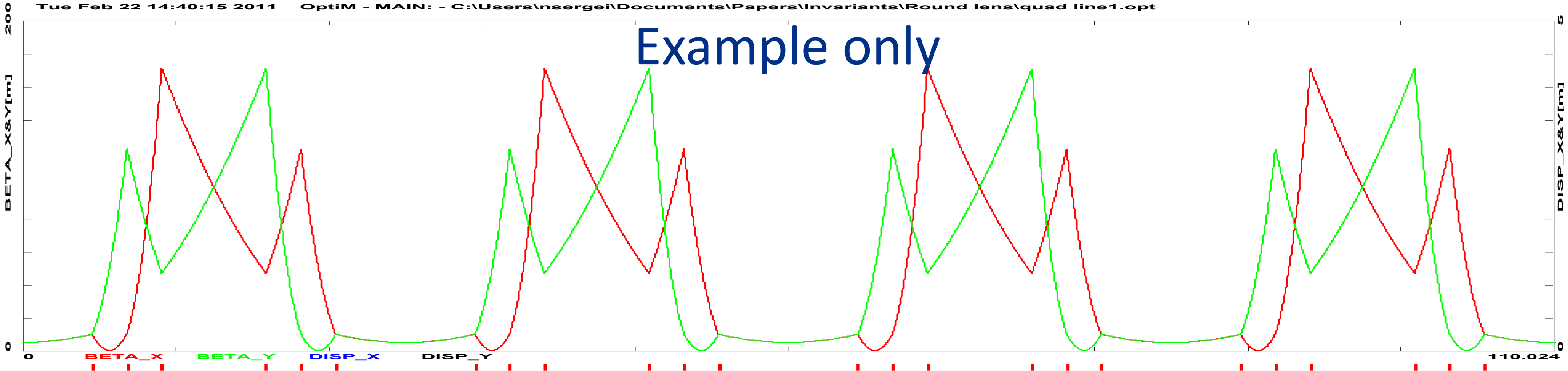
Implementation

1. Start with a round axially-symmetric *linear* lattice (FOFO) with the element of periodicity consisting of
   a. Drift L
   b. Axially-symmetric focusing block “T-insert” with phase advance n × π



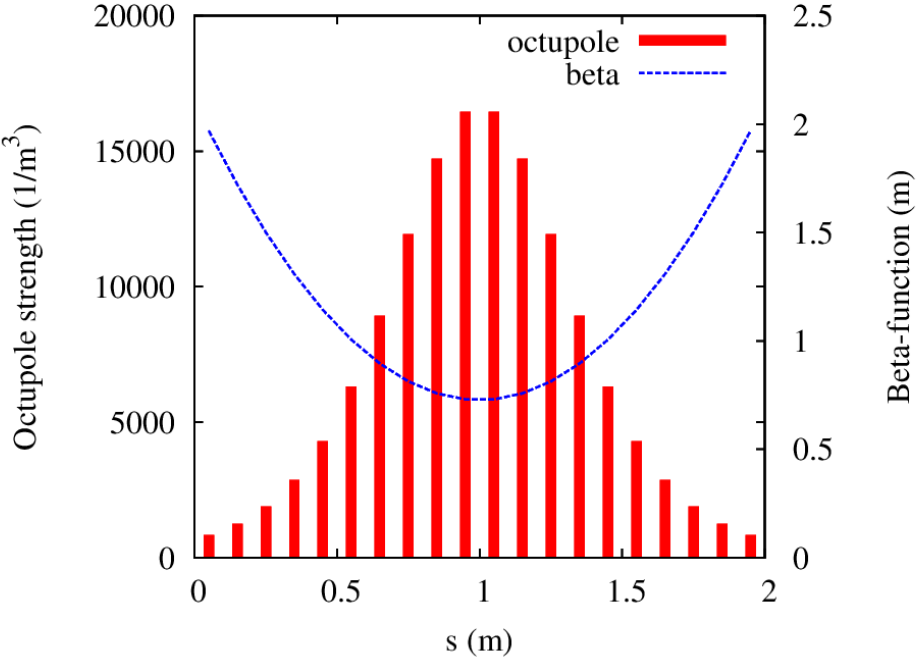
2. Add special nonlinear potential *V(x,y,s)* in the drift such that

$$\Delta V(x,y,s) \approx \Delta V(x,y) = 0$$

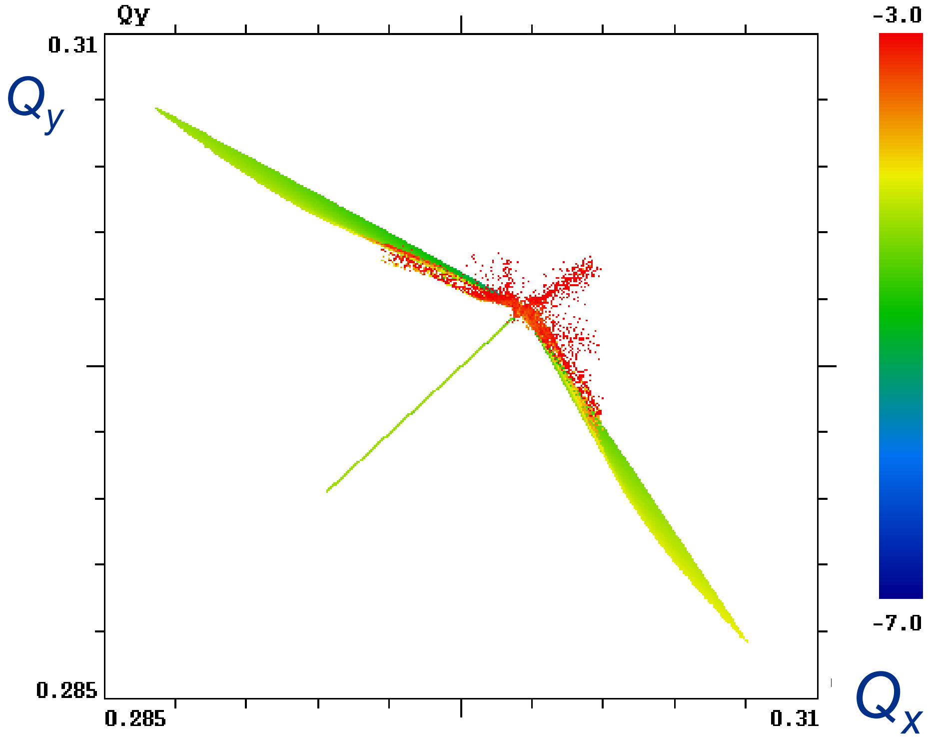


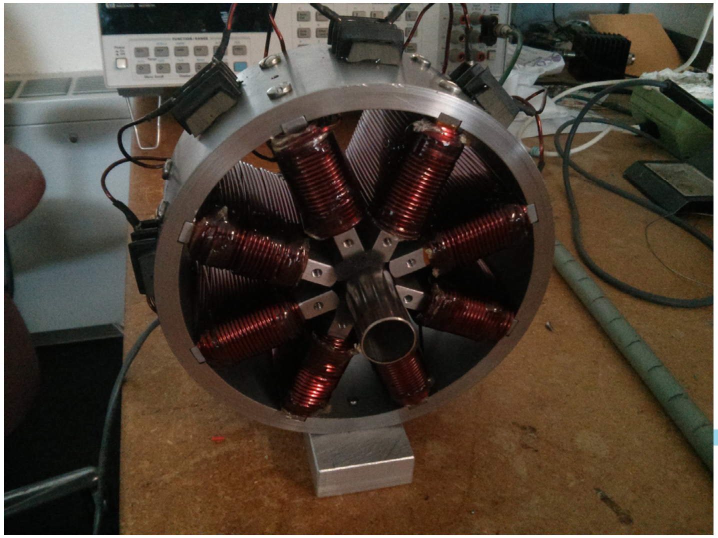
Example only

Octupoles



20 octupoles, scaled as $1/\beta(s)^3$





- While the dynamic aperture is limited, the attainable tune spread is large ~0.03 – compare to 0.001 created by LHC octupoles

Example 3 (Phys. Rev. ST Accel. Beams 13, 084002)

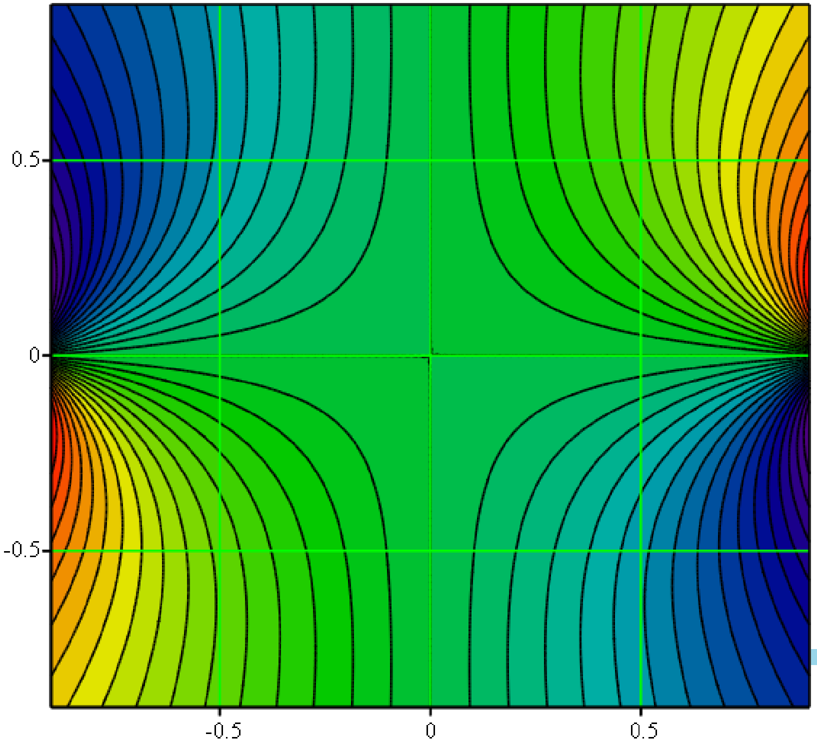
- An integrable nonlinear system with a special Darboux potential (separable in elliptic coordinates).

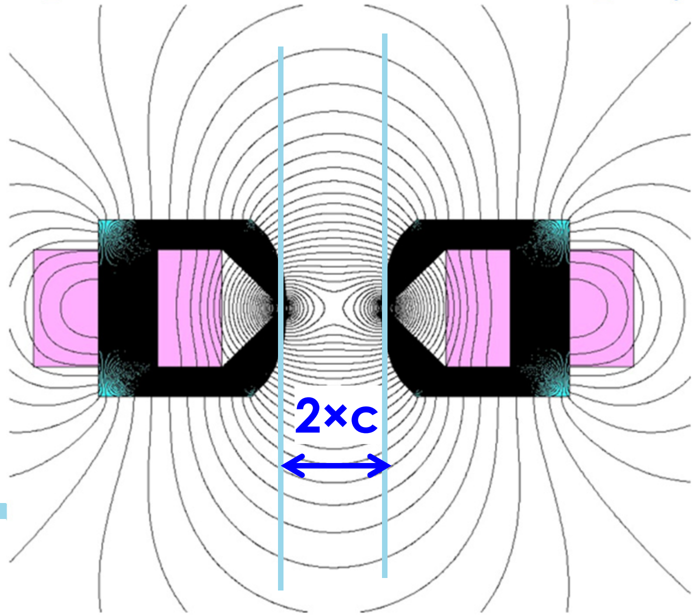
$$H = \frac{1}{2}(p_{nx}^2 + p_{ny}^2) + \frac{1}{2}(x_n^2 + y_n^2) + U(x_n, y_n)$$

$$U(x,y) \approx \frac{t}{c^2}\,\mathrm{Im}\left((x+iy)^2 + \frac{2}{3c^2}(x+iy)^4 + \frac{8}{15c^4}(x+iy)^6 + \frac{16}{35c^6}(x+iy)^8 + \ldots \right)$$

For $|z| < c$

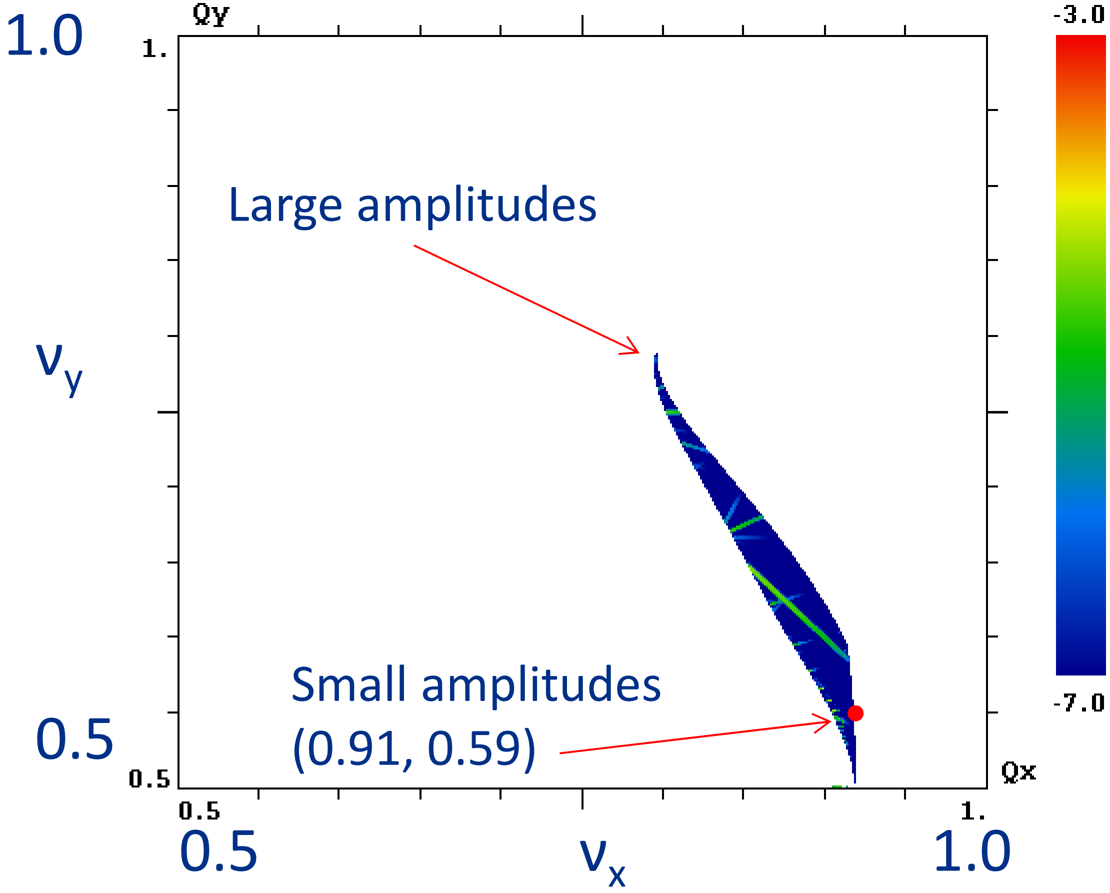
This potential has two adjustable parameters: *t* – strength and *c* – location of singularities





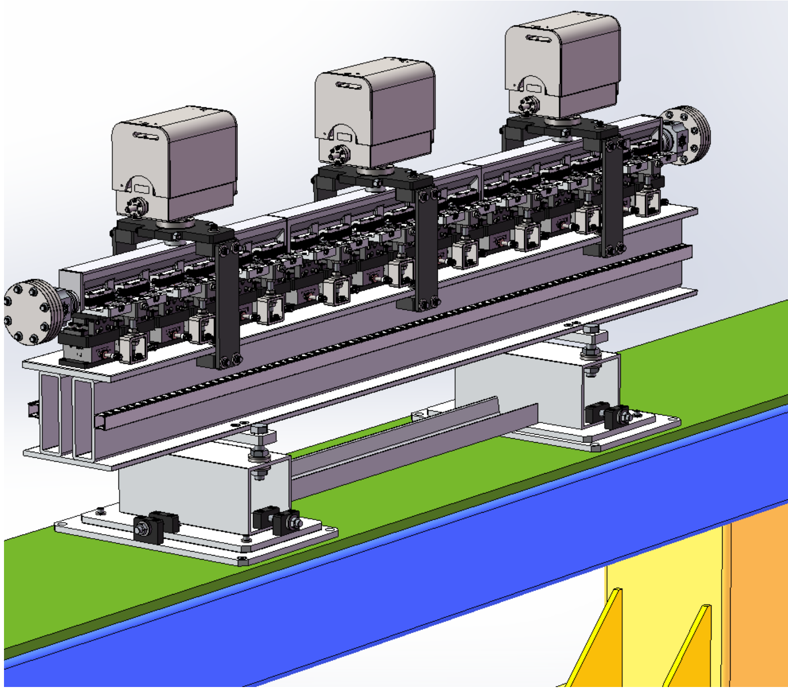
- A single 2-m long nonlinear lens creates a tune spread of ~0.25.

FMA, fractional tunes



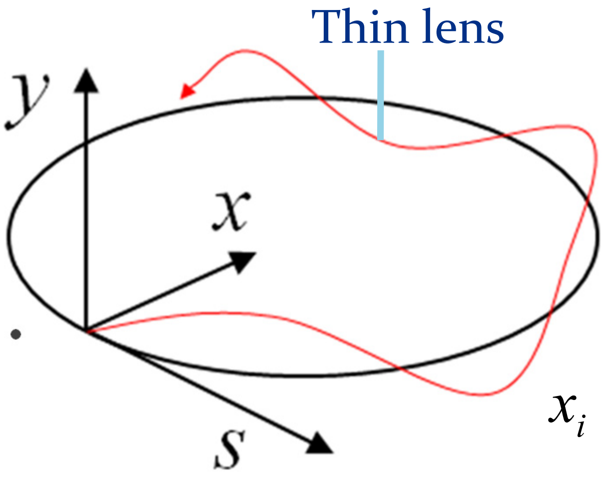
Large amplitudes

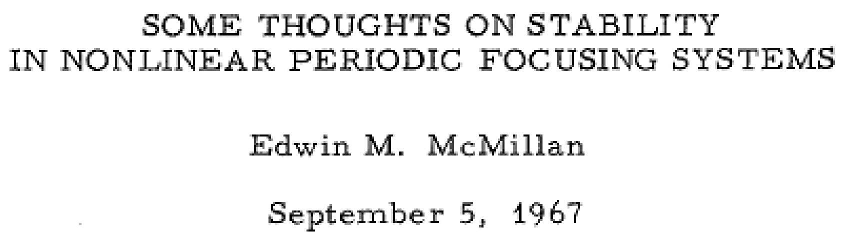
Small amplitudes
(0.91, 0.59)



1.8-m long magnet to be delivered in 2016

Example 4: McMillan mapping




SOME THOUGHTS ON STABILITY
IN NONLINEAR PERIODIC FOCUSING SYSTEMS

Edwin M. McMillan

September 5, 1967

- 1D – thin lens kick

$$f(x) = -\frac{Bx^2 + Dx}{Ax^2 + Bx + C}$$

$$x_i = p_{i-1}$$

$$p_i = -x_{i-1} + f(x_i)$$

$$Ax^2p^2 + B\left(x^2p + xp^2\right) + C\left(x^2 + p^2\right) + Dxp = \text{const}$$

- 2D – a thin lens solution can be carried over to the 2D case in an axially symmetric system

1. The ring with transfer matrix

$$\begin{pmatrix} cI & sI \\ -sI & cI \end{pmatrix} \begin{pmatrix} 0 & \beta & 0 & 0 \\ -\frac{1}{\beta} & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta \\ 0 & 0 & -\frac{1}{\beta} & 0 \end{pmatrix}$$

$$c = \cos(\phi)$$

$$s = \sin(\phi)$$

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

2. Axially-symmetric thin kick

$$\theta(r) = \frac{kr}{ar^2 + 1}$$

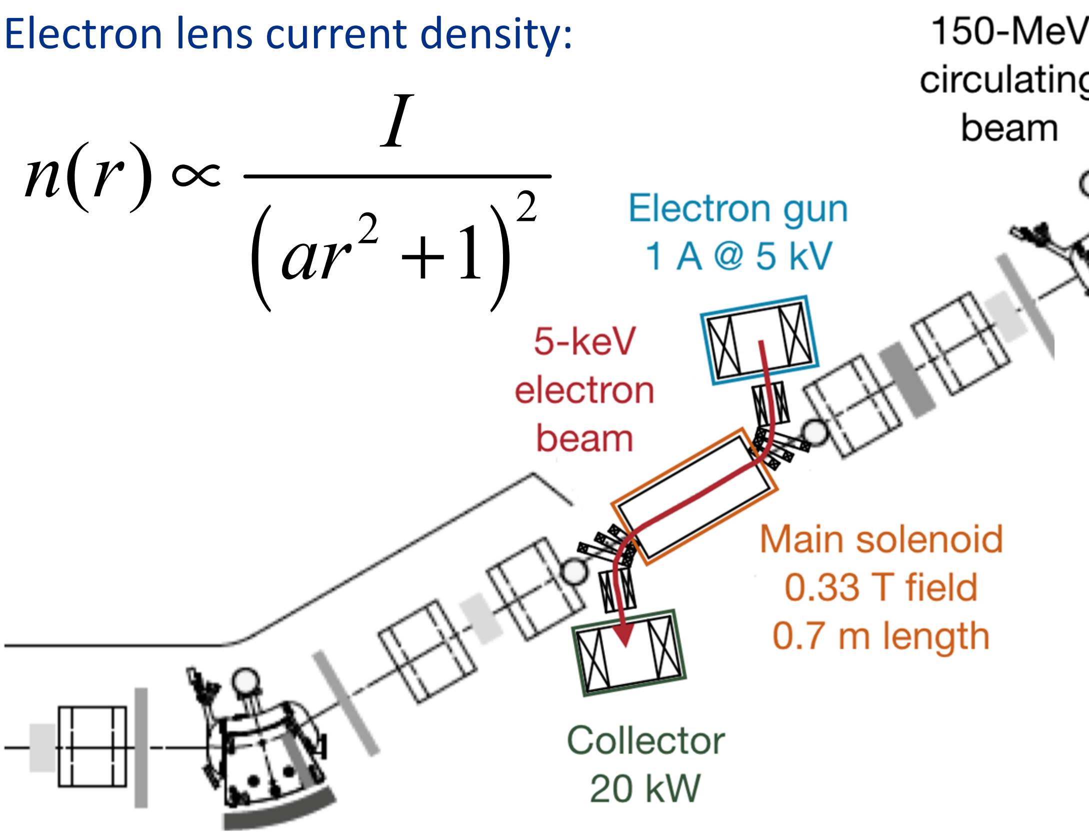
can be created with an electron lens

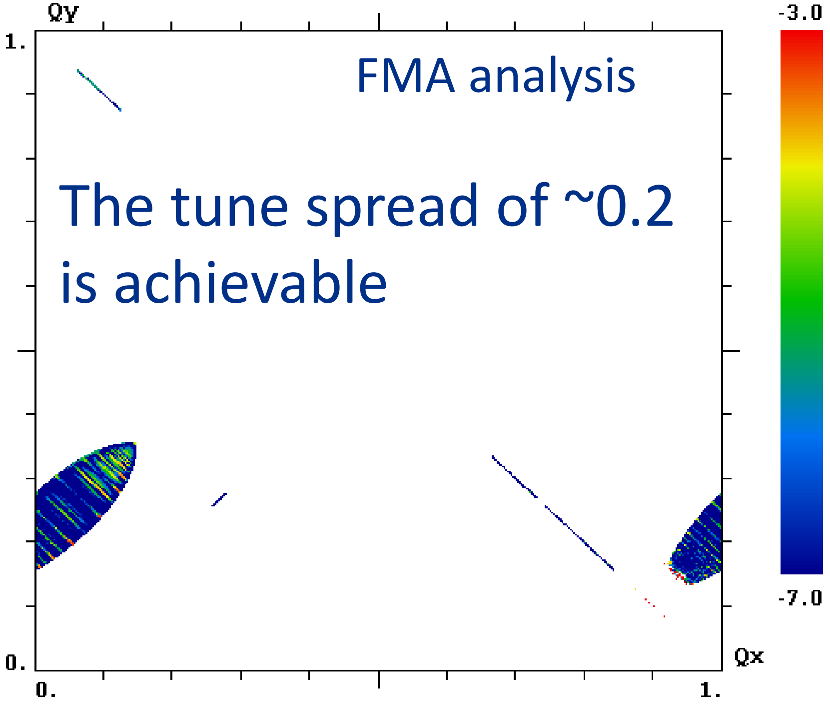
McMillan electron lens

- Capitalize on the Tevatron experience and recent LARP work
- Re-use Tevatron EL components

Electron lens current density:

$$n(r) \propto \frac{I}{\left(ar^2+1\right)^2}$$





FMA analysis

The tune spread of ~0.2 is achievable

G. Stancari, poster on Mon

Enter IOTA

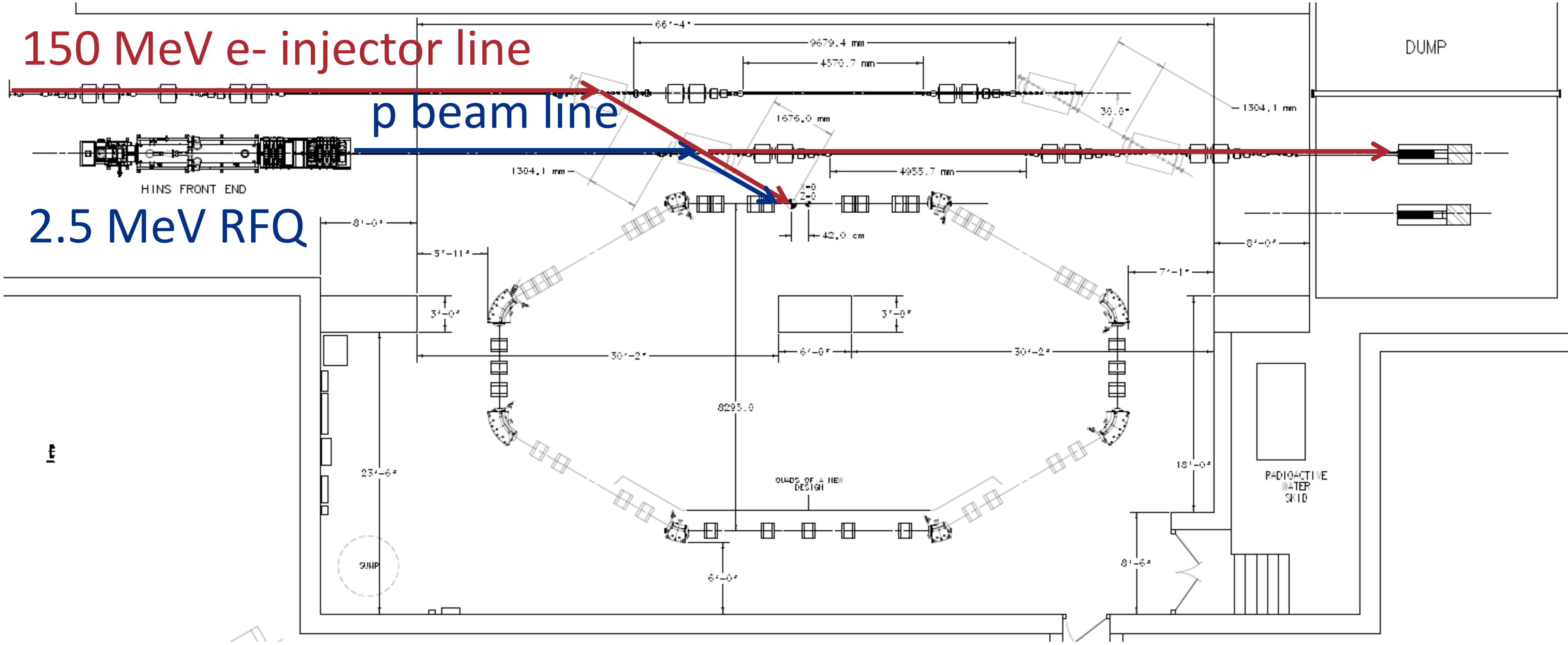
- We have several **innovative ideas for Research**:
  - *Integrable Nonlinear Optics*
  - *Space Charge Compensation*
  - *Optical Stocastic Cooling*
- To test them, we are building the **Integrable Optics Test Accelerator (IOTA)**

- **There are no dedicated ring-based accelerator test facilities** in the US
  - UMER at UMD is operating with 10keV electrons, only few turns

Integrable Optics Test Accelerator

- **Unique features**:
  - Can operate with either electrons or protons (up to 150 MeV/c momentum)
  - Large aperture
  - Significant flexibility of the lattice
  - Precise control of the optics quality and stability
  - Very-high intensity operations (with protons)
- **Based on conventional technology** (magnets, RF)
- **Cost-effective solution**
  - Balance between low energy (low cost) and research potential

IOTA Ring



150 MeV e- injector line

p beam line

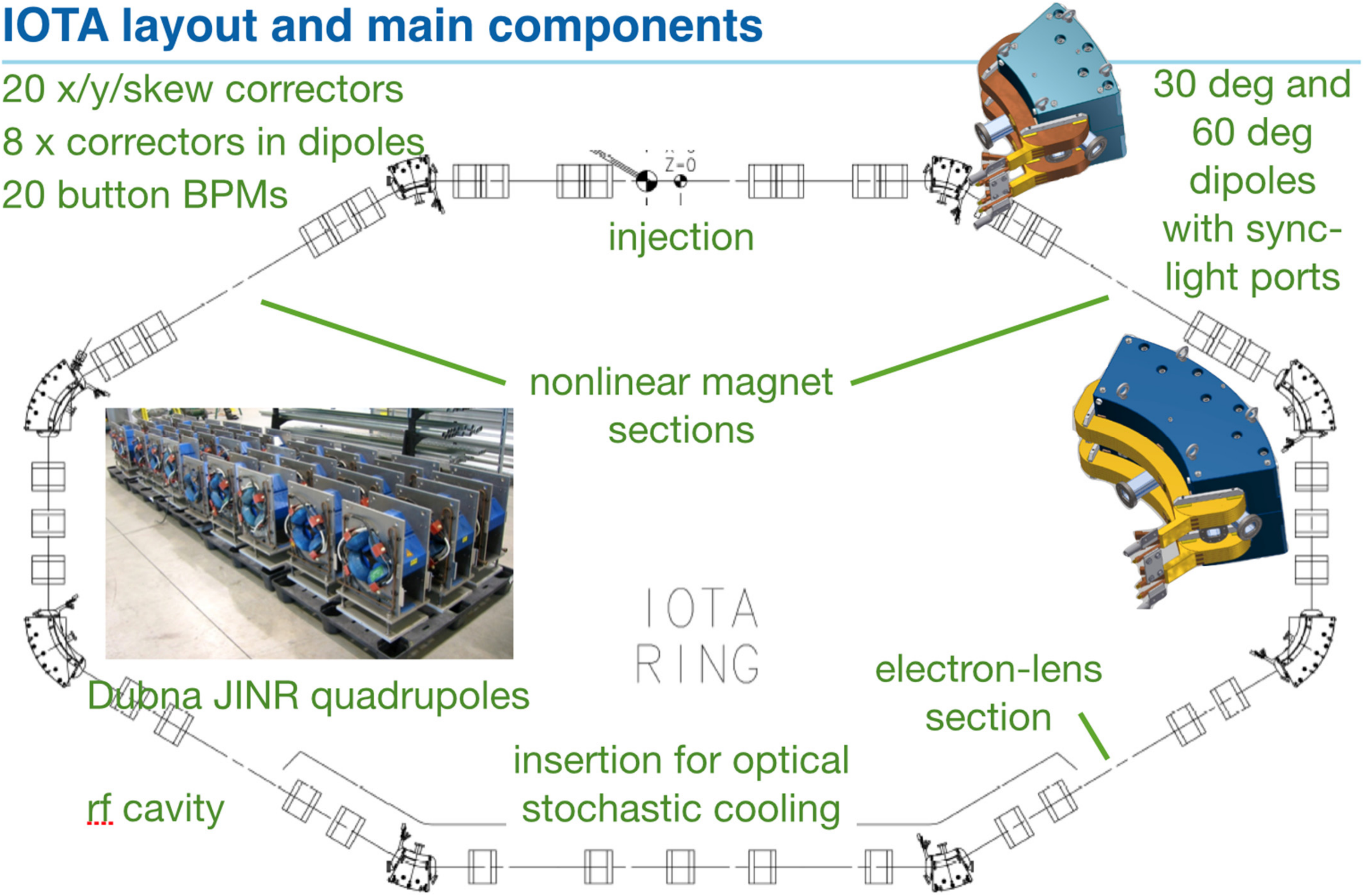
2.5 MeV RFQ

IOTA layout and main components

20 x/y/skew correctors

8 x correctors in dipoles

20 button BPMs



injection

30 deg and 60 deg dipoles with sync-light ports

nonlinear magnet sections

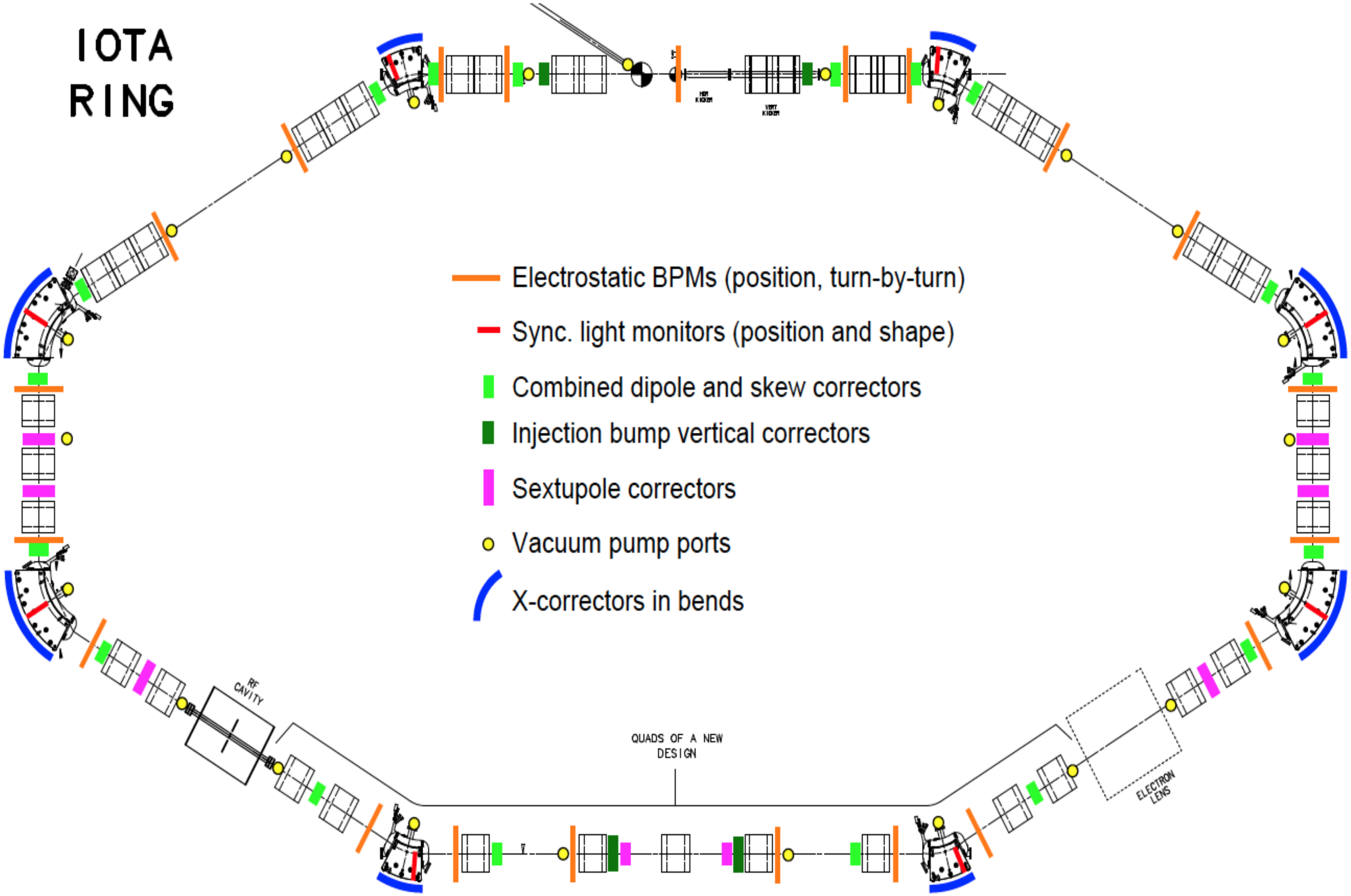
Dubna JINR quadrupoles

electron-lens section

insertion for optical stochastic cooling

rf cavity

IOTA Layout

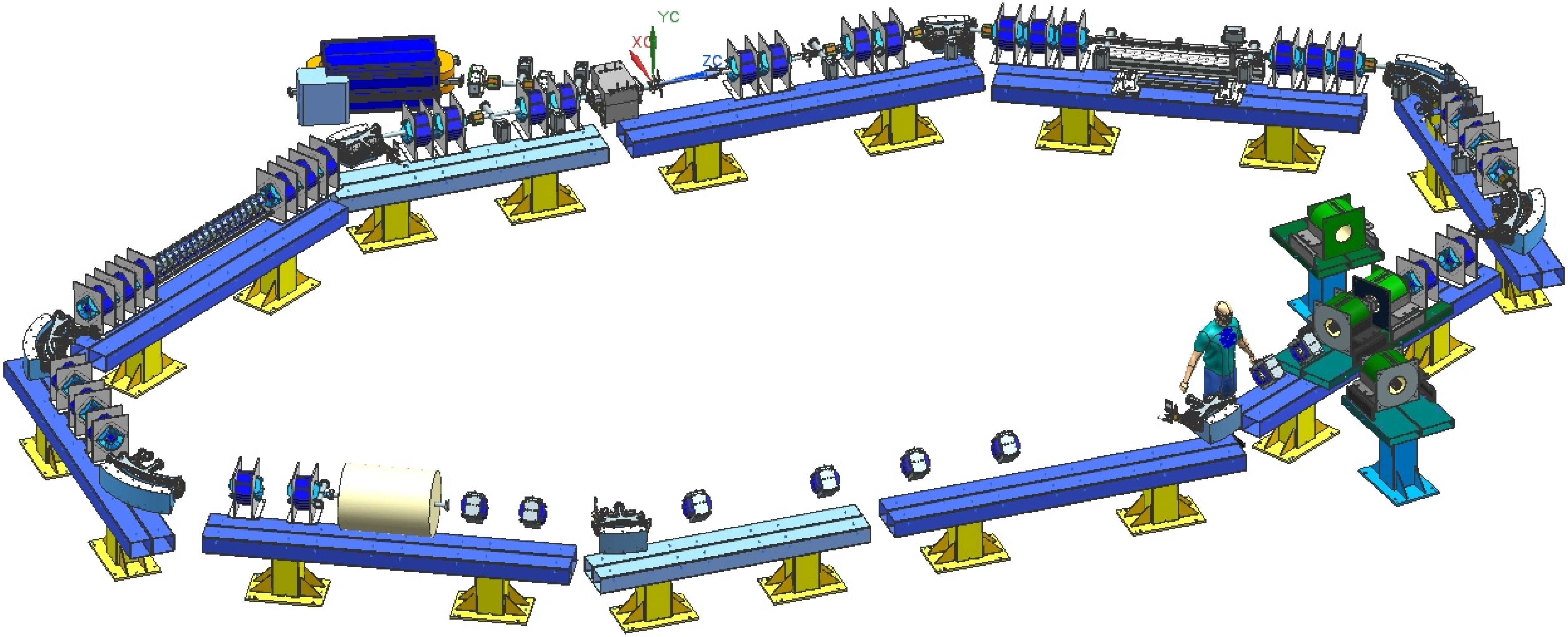


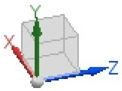
- — Electrostatic BPMs (position, turn-by-turn)
- — Sync. light monitors (position and shape)
- ▮ Combined dipole and skew correctors
- ▮ Injection bump vertical correctors
- ▮ Sextupole correctors
- ● Vacuum pump ports
- (X-correctors in bends

IOTA Parameters

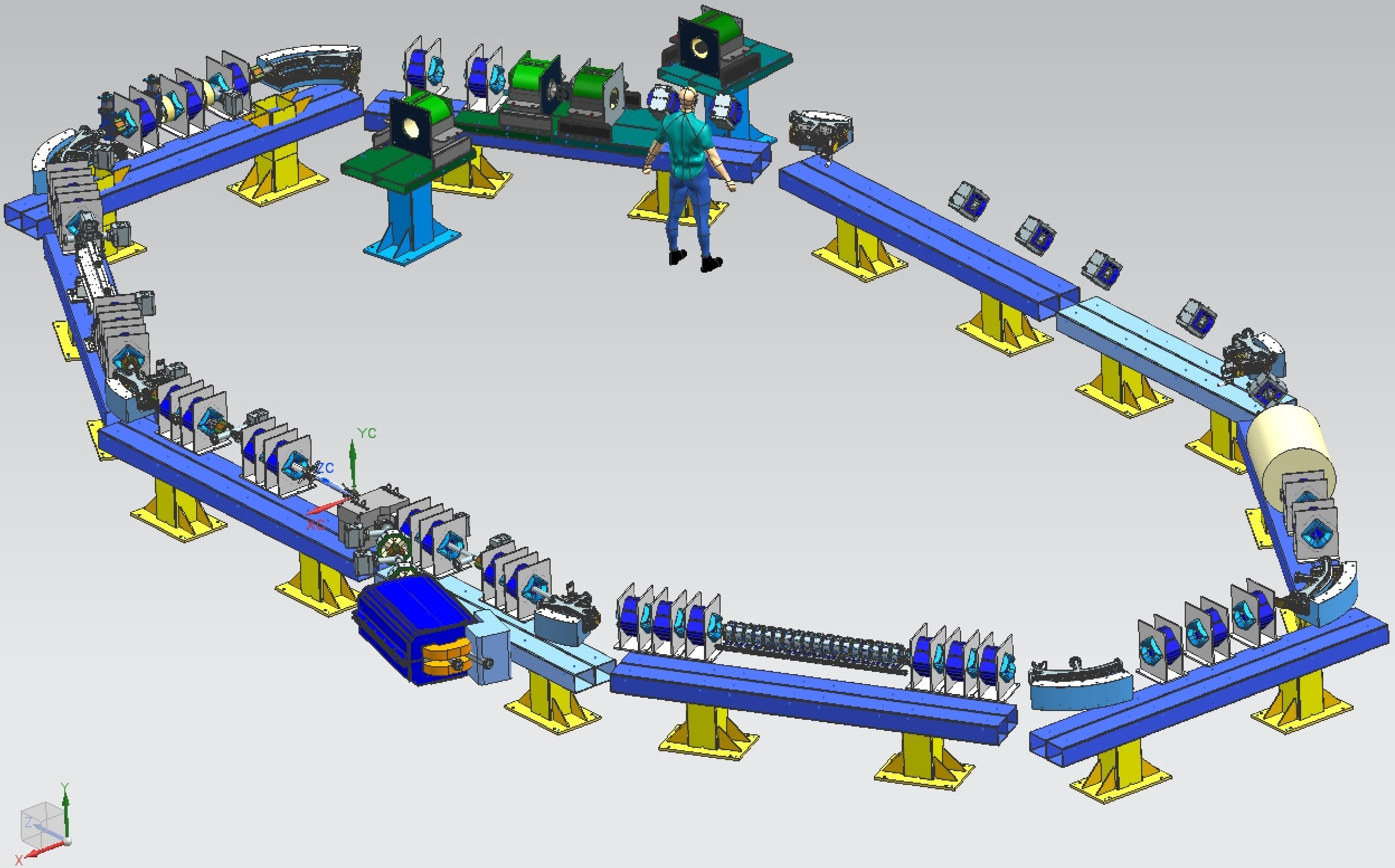
Nominal kinetic energy	e^-: 150 MeV, p+: 2.5 MeV
Nominal intensity	e^-: 1×10^9, p+: 1×10^{11}
Circumference	40 m
Bending dipole field	0.7 T
Beam pipe aperture	50 mm dia.
Maximum b-function (x,y)	12, 5 m
Momentum compaction	0.02 ÷ 0.1
Betatron tune (integer)	3 ÷ 5
Natural chromaticity	-5 ÷ -10
Transverse emittance r.m.s.	e^-: 0.04 μm, p+: 2μm
SR damping time	0.6s (5×10^6 turns)
RF V,f,q	e^-: 1 kV, 30 MHz, 4
Synchrotron tune	e^-: 0.002 ÷ 0.005
Bunch length, momentum spread	e^-: 12 cm, 1.4×10^{-4}

IOTA Layout

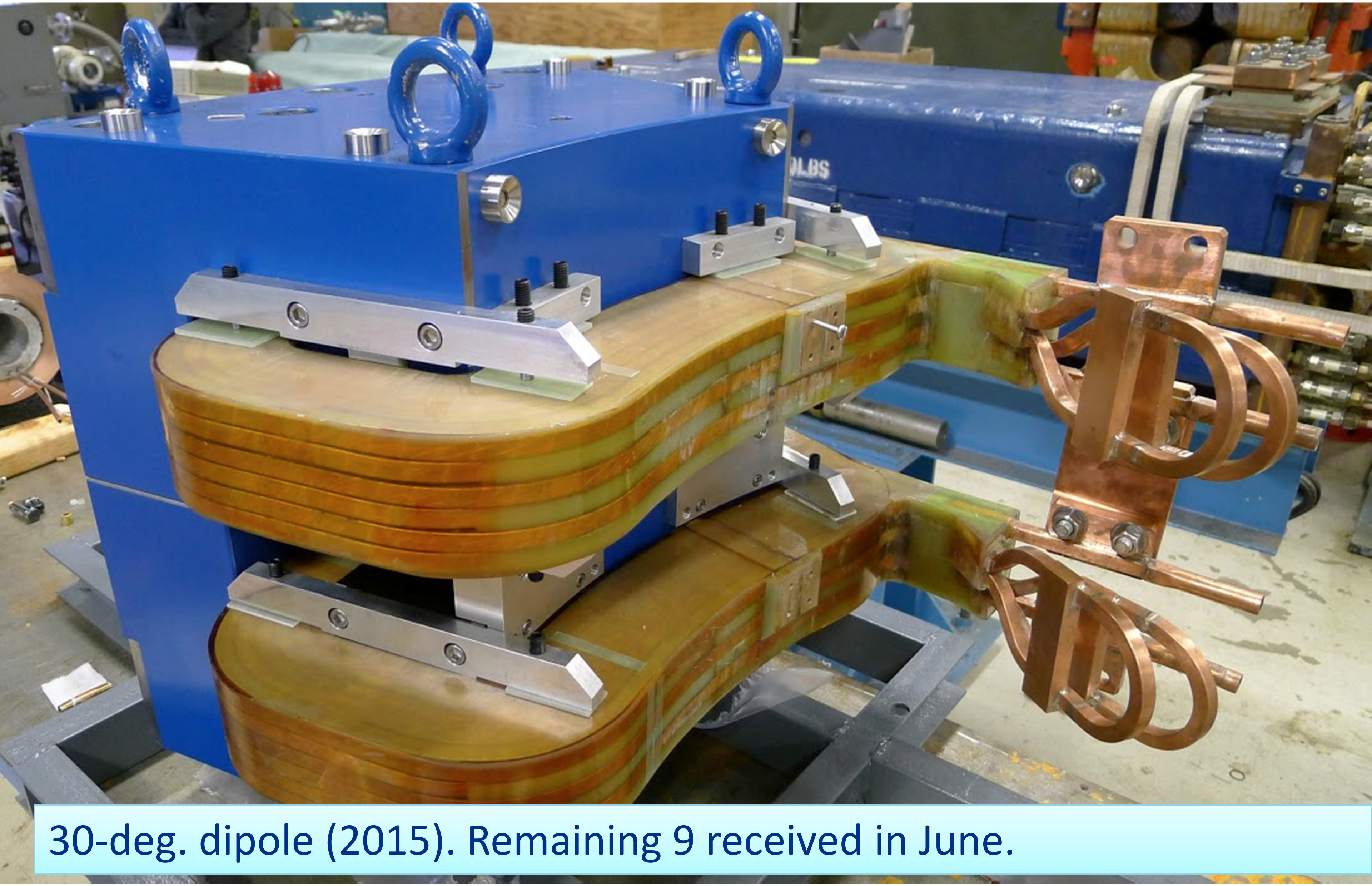




IOTA Layout



Hall cleaned, cable trays installed.



30-deg. dipole (2015). Remaining 9 received in June.

Major IOTA Milestones:

- FY16:
  - 50 MeV e- beam injector (done)
  - Finish installation of high energy beamline from CM to beam dump
  - IOTA ring components 80% procured
- FY17:
  - 300 MeV beam to beam dump
  - Procure the remaining 20% of IOTA components and install IOTA
- FY18:
  - 150 MeV e- beam injected in IOTA
  - Finish IOTA commissioning and start research (NL-IO)
- FY19:
  - Move the 2.5 MeV-proton RFQ to the IOTA building
  - Commission the IOTA proton injection, so that the research with protons can start in FY20

Summary

We have a very exciting IOTA research program centered around nonlinear beam dynamics and advanced beam cooling

1. **Nonlinear Integrable Optics**
2. **Space Charge Compensation**
3. **Optical Stochastic Cooling** – Proof-of-principle demonstration
4. **Beam collimation –** Technology development for a hollow electron beam collimation system
5. **Electron Cooling –** Advanced techniques

- First IOTA experiments on nonlinear integrable optics with e- is in a good shape
  - Simulations done; nonlinear magnets on track