Simulation of Space-Charge Compensation of a Low-Energy Proton Beam in a Drift Section

Daniel Noll

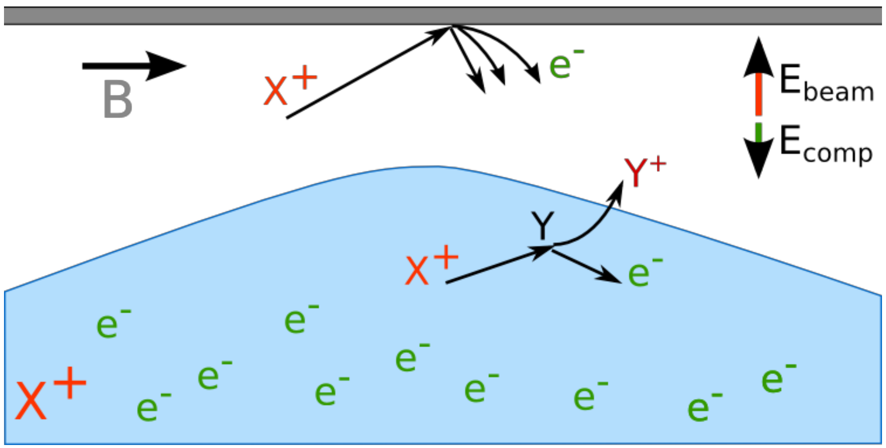
Institute for Applied Physics
Goethe University Frankfurt am Main

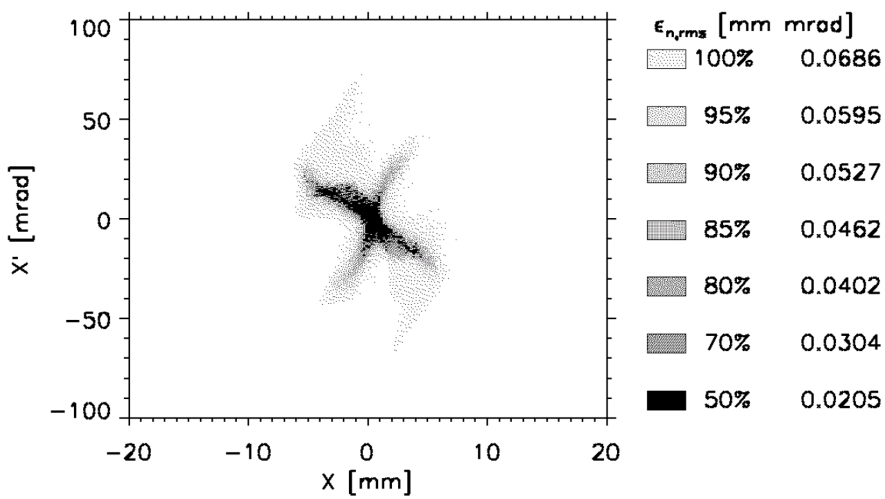
57th ICFA Advanced Beam Dynamics Workshop on High-Intensity and High-Brightness Hadron Beams
Malmö, Sweden, 3-8 July 2016

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

IAP Institut für Angewandte Physik Frankfurt am Main

Space-Charge Compensation





Measured beam distribution after compensated transport through 2 solenoids [1]

- Accumulation of secondary particles of opposite charge in the beam potential
- “Traditional” treatment: Constant compensation factor
- Include secondary particles in self-consistent simulation

(Computational) challenges

- Long simulation times

$$t_{\text{Compensation}} = \frac{kT}{vp\sigma} = 17\mu s$$

120 keV p^+, N_2, p=10^{-3} Pa

- Magnetic fields

$$t_{\text{cyclotron}} = \frac{2\pi m}{qB} = 71\ ps,\ \ B = 0.5\ T$$

- Which effects to include?

[1] P. Groß, Untersuchungen zum Emittanzwachstum intensiver Ionenstrahlen bei teilweiser Kompensation der Raumladung, Dissertation, Frankfurt 2000

Outline

- Motivation
- Simulation model
- Results for a drift section

  Time development, Charge densities, Velocity distributions
- … how to get them by simpler means

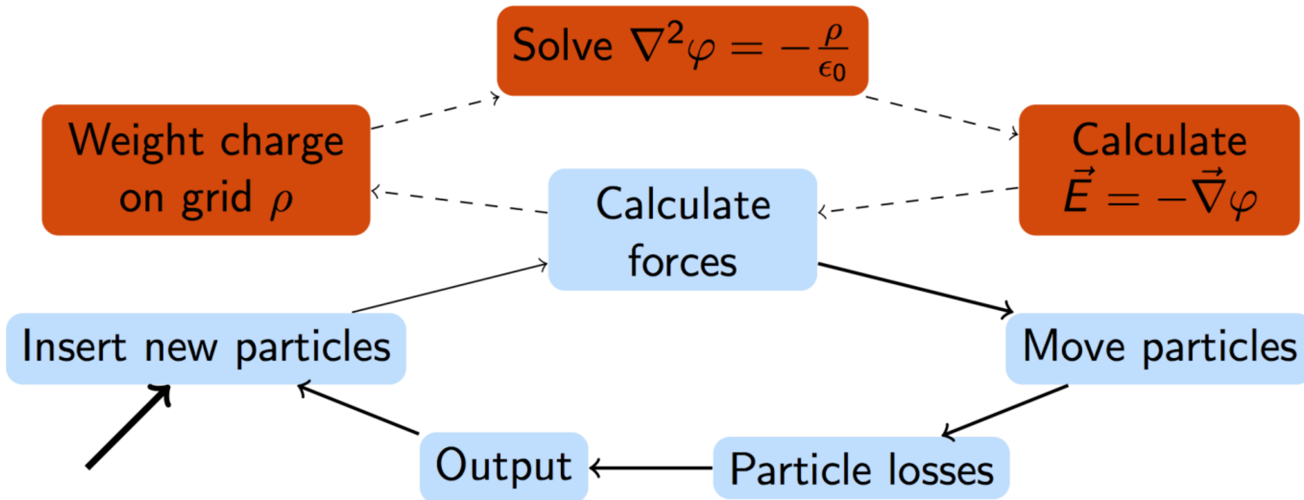
  Poisson-Boltzmann equation
- … and what is wrong with them

  Stochastical heating

Simulation model

Solution of the Vlasov-Poisson system by introducing simulation particles

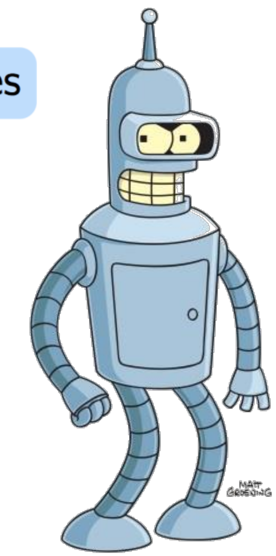
$$\frac{\partial f}{\partial t} + \frac{\mathbf{p}}{m}\frac{\partial f}{\partial q} + qE\frac{\partial f}{\partial p} = 0$$



Code used: *bender* [1]

Null collision model [2] using single-differential cross sections for proton & electron impact ionization [3,4]

FFT or finite difference 3d space charge solver / Verlet integration



[1] D. Noll, M. Droba, O. Meusel, U. Ratzinger, K. Schulte, C. Wiesner – The Particle-in-Cell Code bender and Its Application to Non-Relativistic Beam Transport, HB2014, WEO4LR02.
[2] Rudd, Kim, Madison, Gay - Electron production in proton collisions with atoms and molecules: energy distributions, Rev. Mod. Phys. 64, 441-490 992).
[3] Kim, Rudd – Binary-Encounter-Dipole Model for Electron-Impact Ionization, Physical Review A, 50(5), 3954.
[4] Vahedi, Surendra – A Monte Carlo Collision Model for the Particle-in-Cell method, Computer Physics Communications (1995).

Space-Charge Compensation

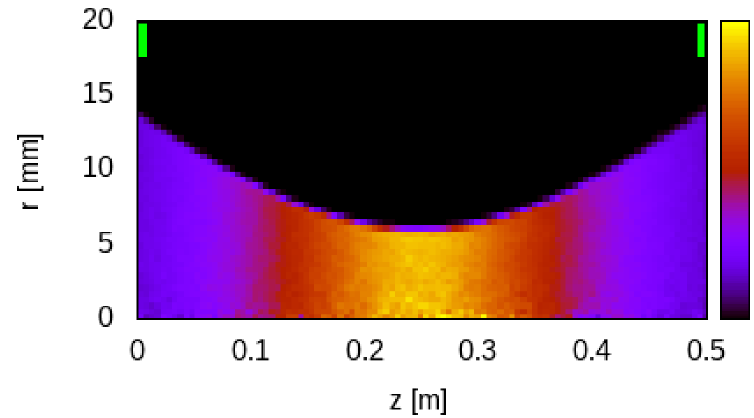
Model system

- Which system to simulate? Should be as simple as possible:
  - Drift section: no magnetic fields
  - No particle losses
  - Argon as residual gas
    - High ionization cross section
    - No dissociation fragments
    - Good data availability

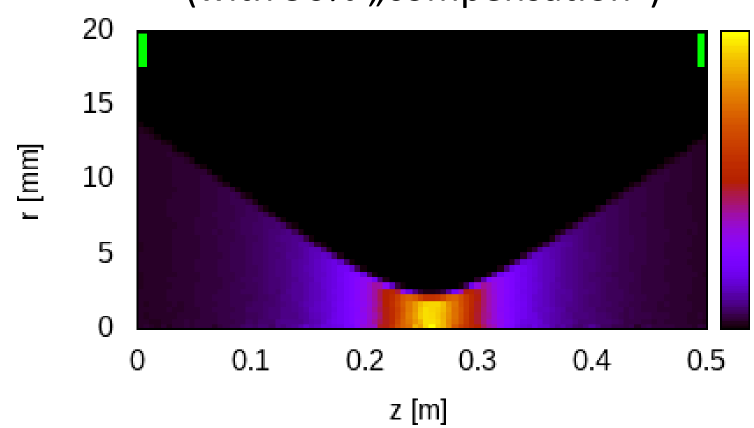
100 mA, 120 keV proton beam
10^{-5} mbar Argon background
-1500 V repeller voltage
$E_{rms,norm}$ = 0.4 mm mrad, α=7.4, β=1.89 m

1000 macroparticles per step
0.4 mm mesh resolution
50 ps time step

Proton density without compensation

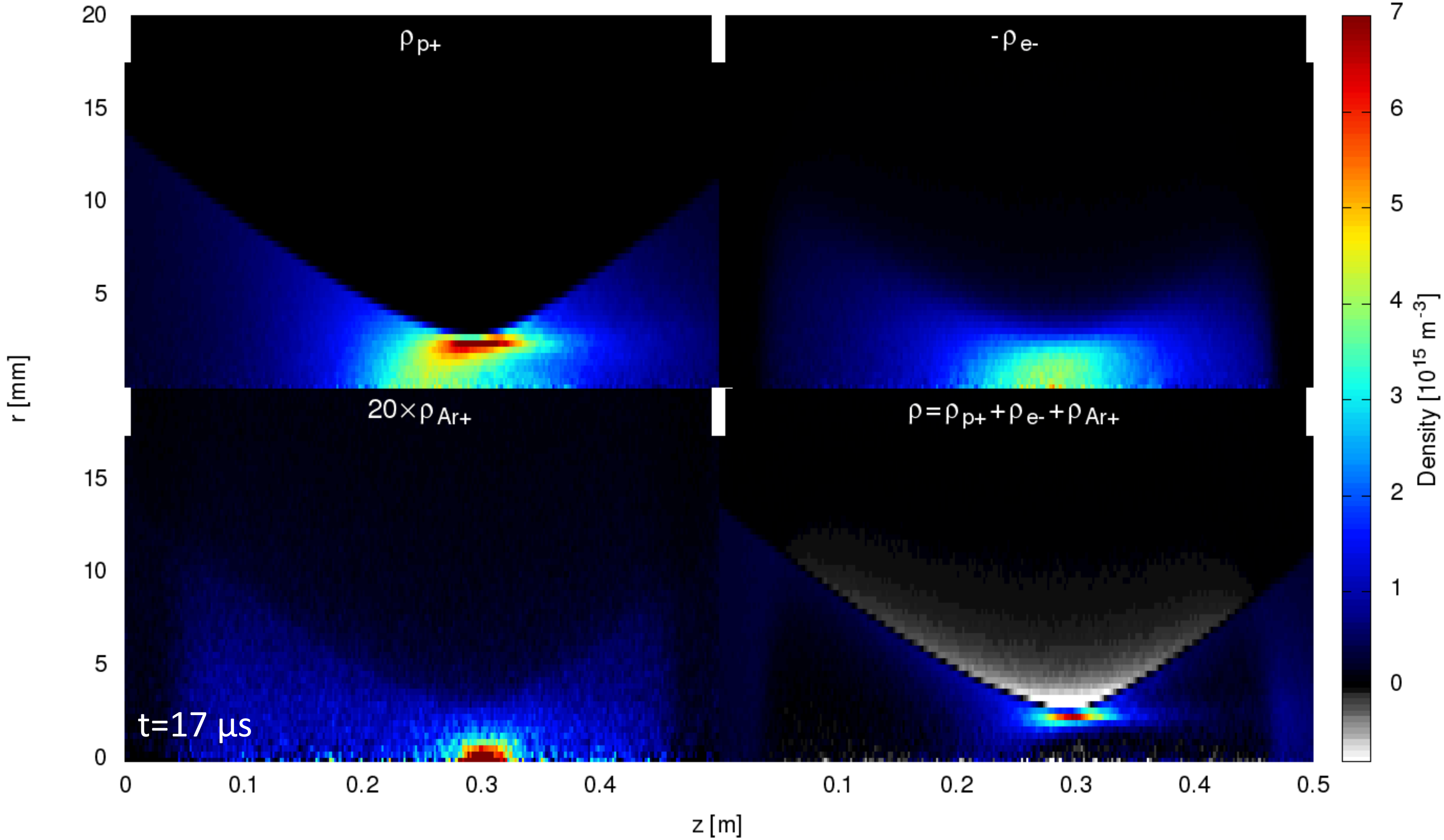


Proton density at 10 mA
(with 90% „compensation“)



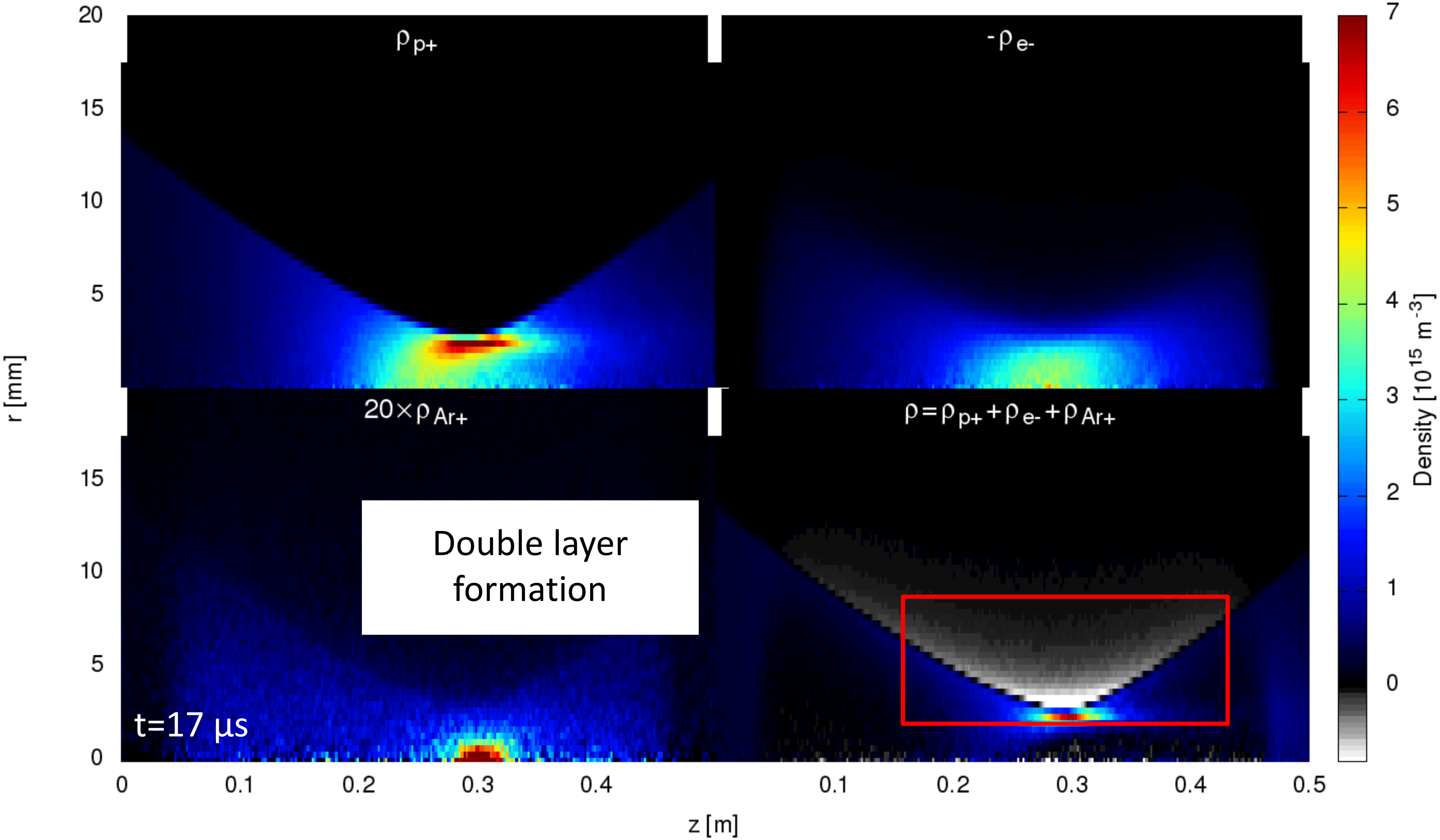
Results for the Drift System

Charge densities



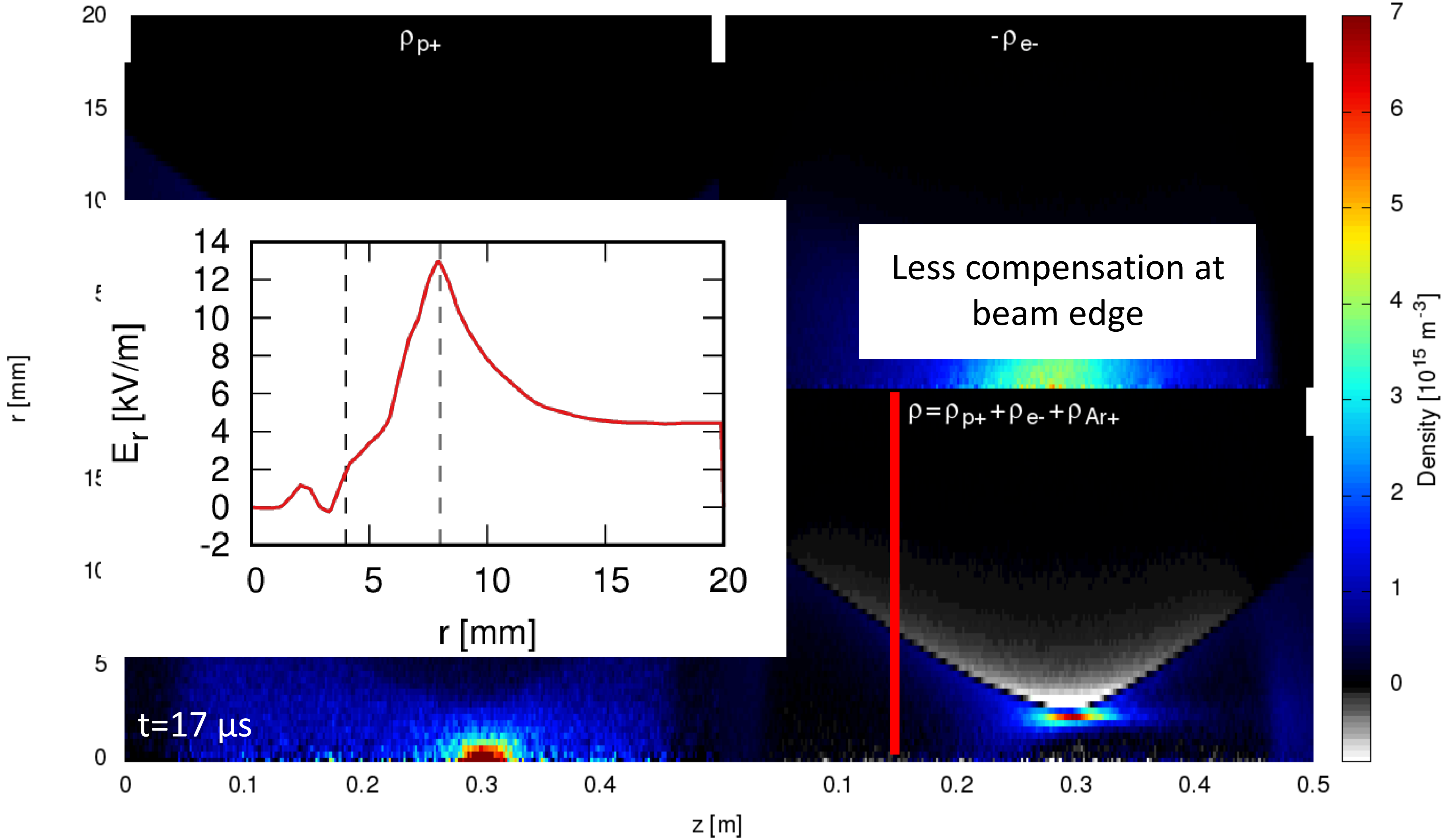
Results for the Drift System

Charge densities



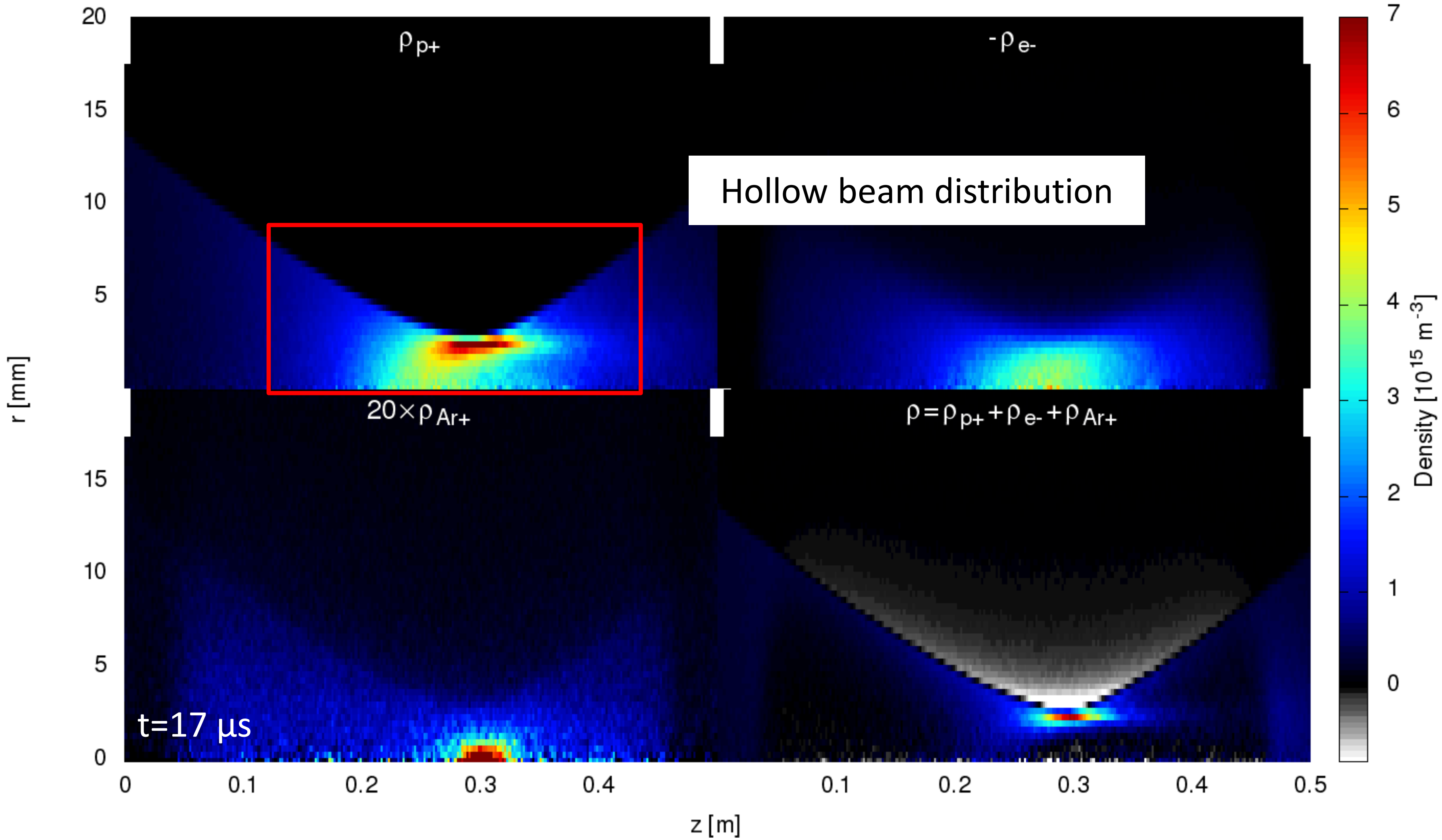
Results for the Drift System

Charge densities



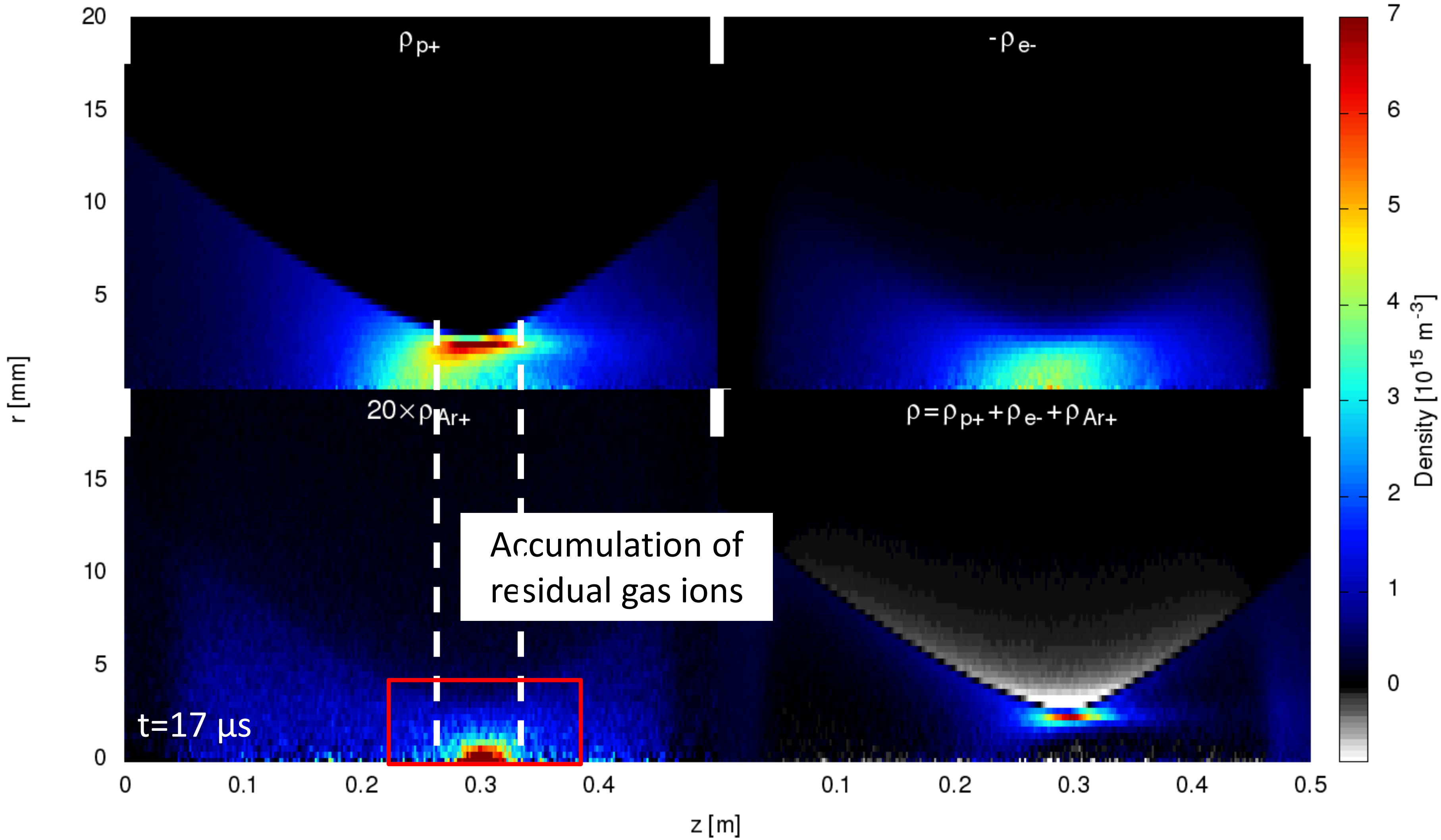
Results for the Drift System

Charge densities



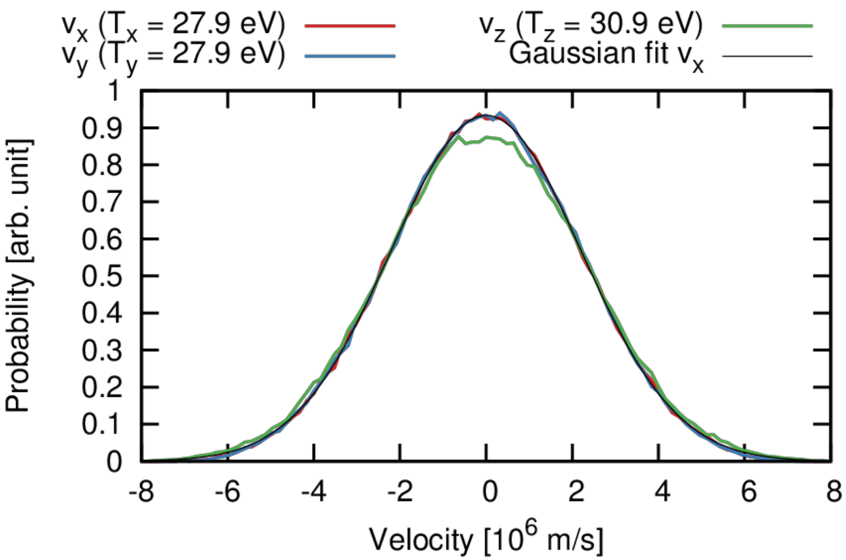
Results for the Drift System

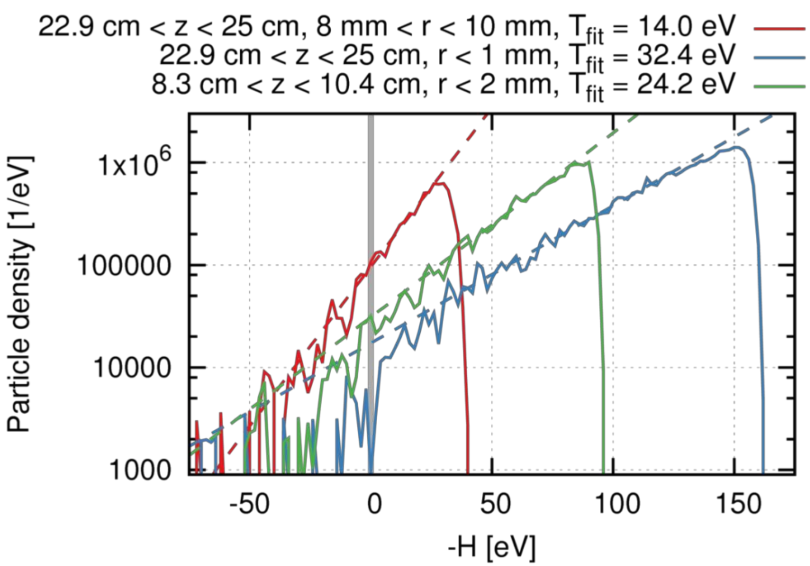
Charge densities



Results for the Drift System

Velocity distribution

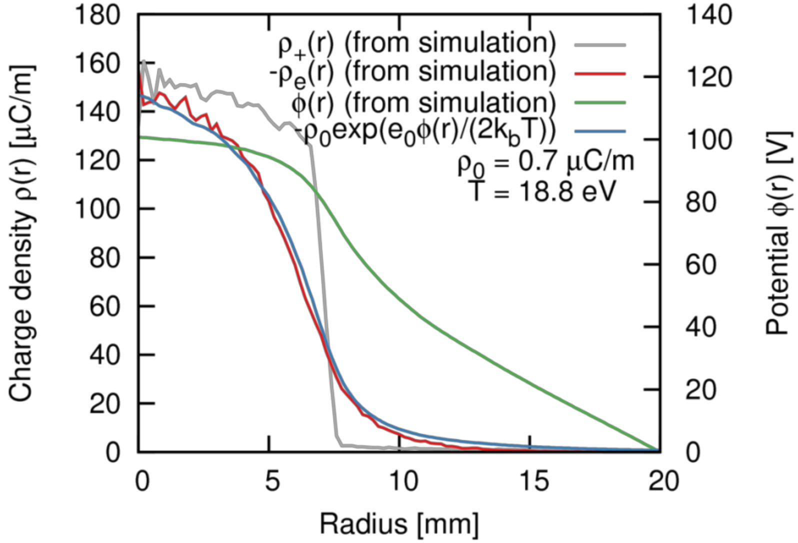




- Gaussian velocity distributions everywhere
  - $T_{x,y} \neq T_z$
  - $T_{x,y} = T_{x,y}(r, z)$
  - Deviation from Gaussians for large radii
  - Remain constant in equilibrium
- Approximately follow a Boltzmann distribution

$$f(\mathbf{r}, \mathbf{p}) = f_0 \exp\left(-\frac{H}{k_\mathrm{b} T}\right)$$

Poisson-Boltzmann Model



- Radial distribution: $f(r) = \widetilde{f}_0 \exp(-e\varphi(r)/kT)$
- T, ρ_0 determine distribution
- Compensation electrons behave like a **non-neutral plasma** confined in the beam potential.

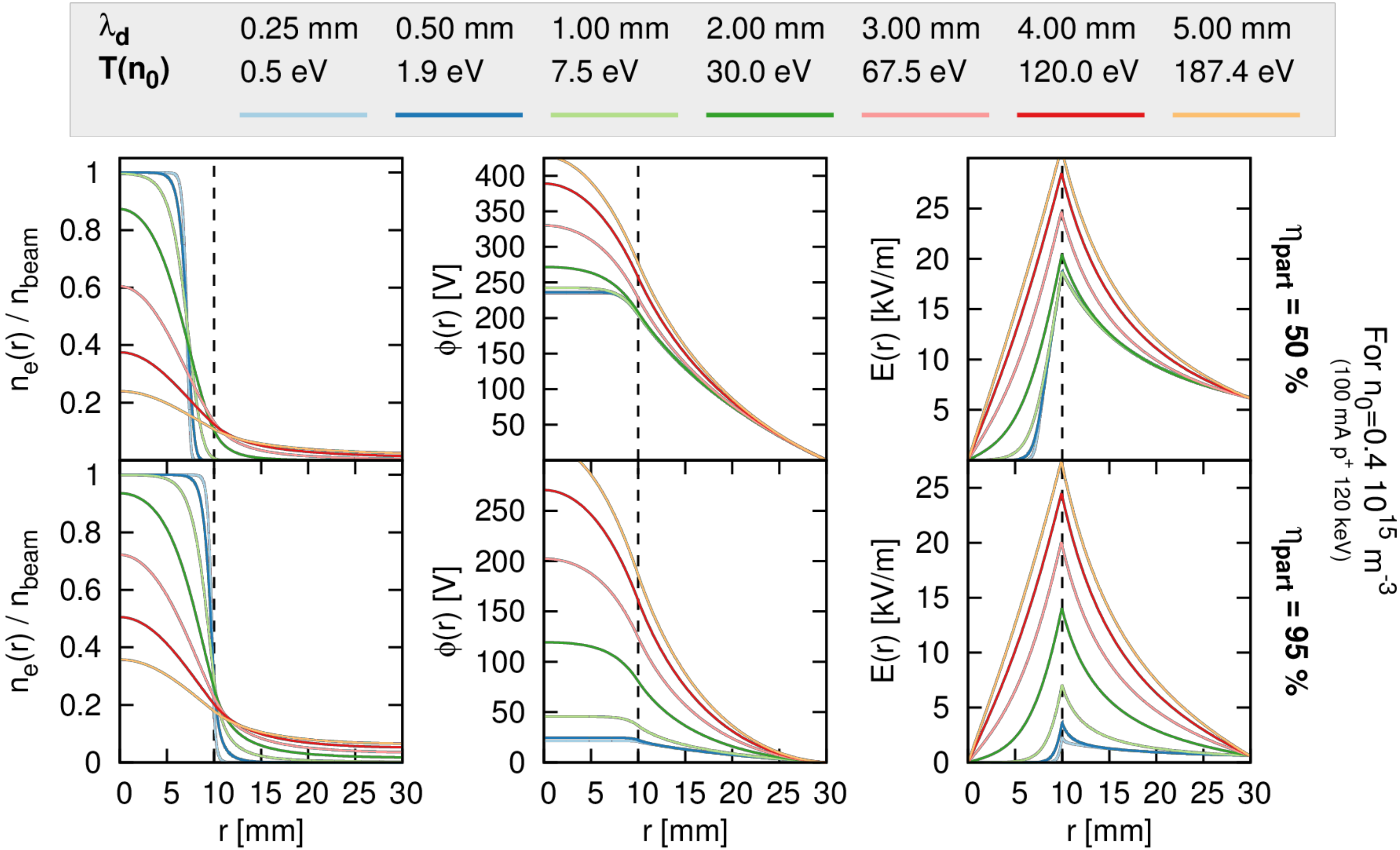
If we know T and ρ_0, can we find φ(r), f(r) directly?

$$f_0 \exp\left(-\frac{\mathbf{p}^2}{2mk_\mathrm{b}T}\right) \exp\left(-\frac{q\left(\varphi_\mathrm{c}(\mathbf{r}) + \varphi_\mathrm{ext}(\mathbf{r})\right)}{k_\mathrm{b}T}\right) \qquad \varphi_\mathrm{c}(\mathbf{r}) + \varphi_\mathrm{ext}(\mathbf{r})$$

$$\nabla^2 \varphi_c(r) = -\frac{q}{\epsilon_0}\int f(\mathbf{r},\mathbf{p})\,\mathrm{d}\mathbf{v} = -\frac{\rho_\mathrm{c}}{\epsilon_0}\exp\left(-\frac{q\varphi(r)}{k_\mathrm{b}T}\right)$$

Poisson-Boltzmann Model

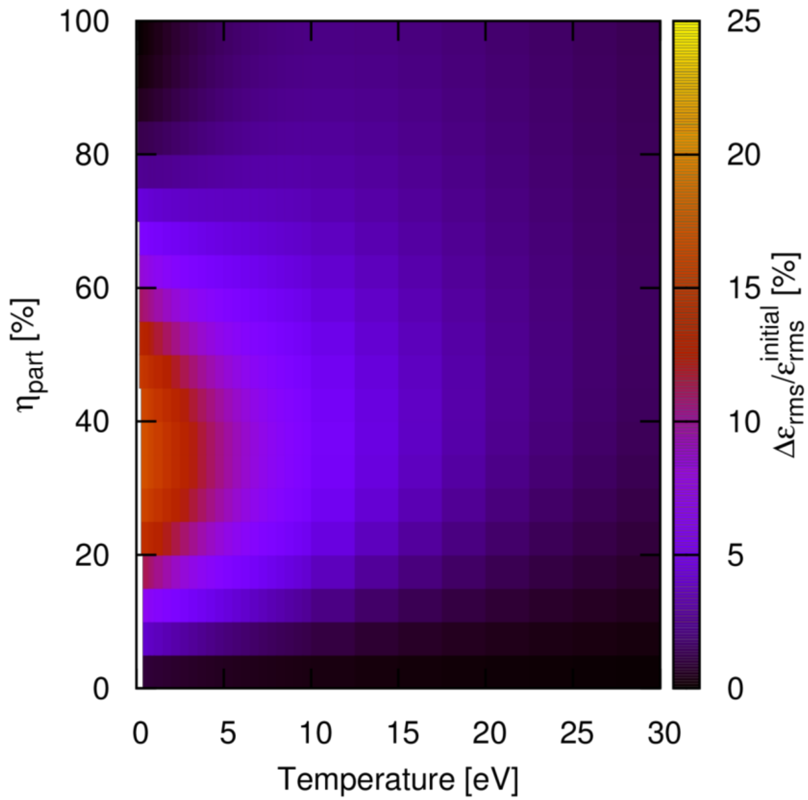
Solutions in r



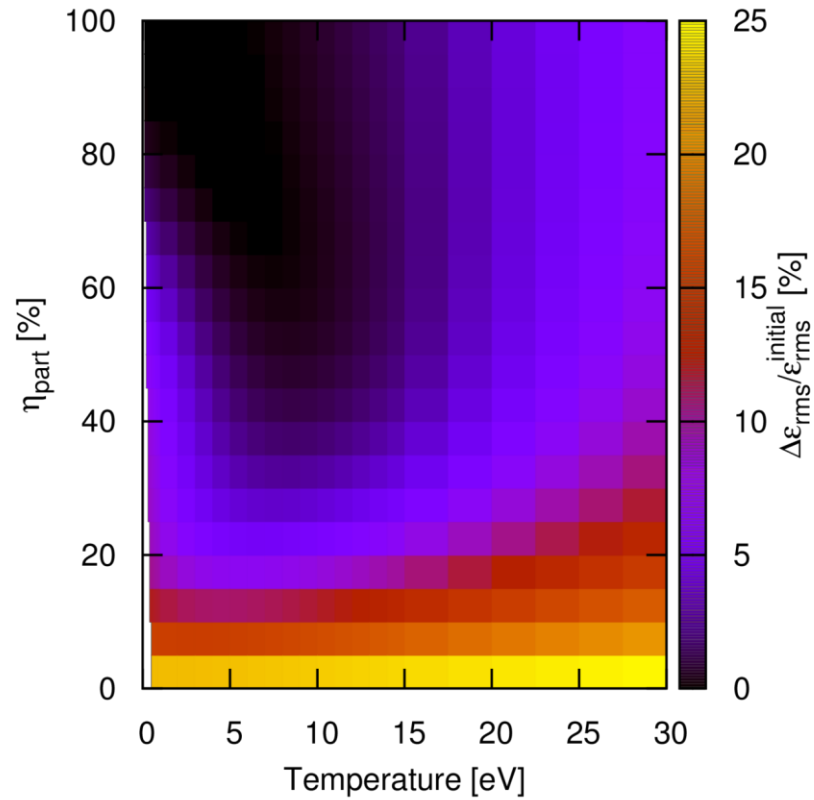
Poisson-Boltzmann Model

Emittance growth

KV distribution



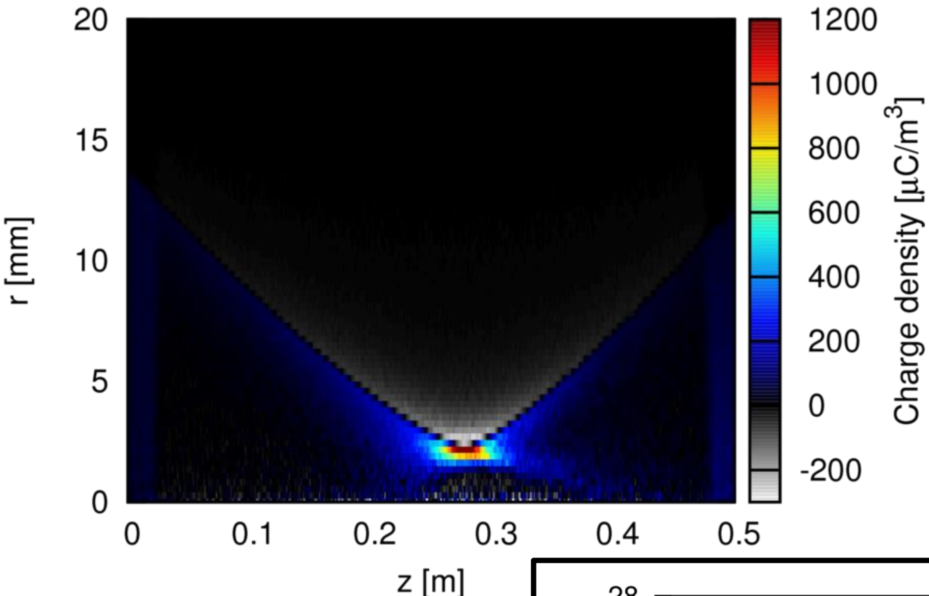
Gaussian distribution

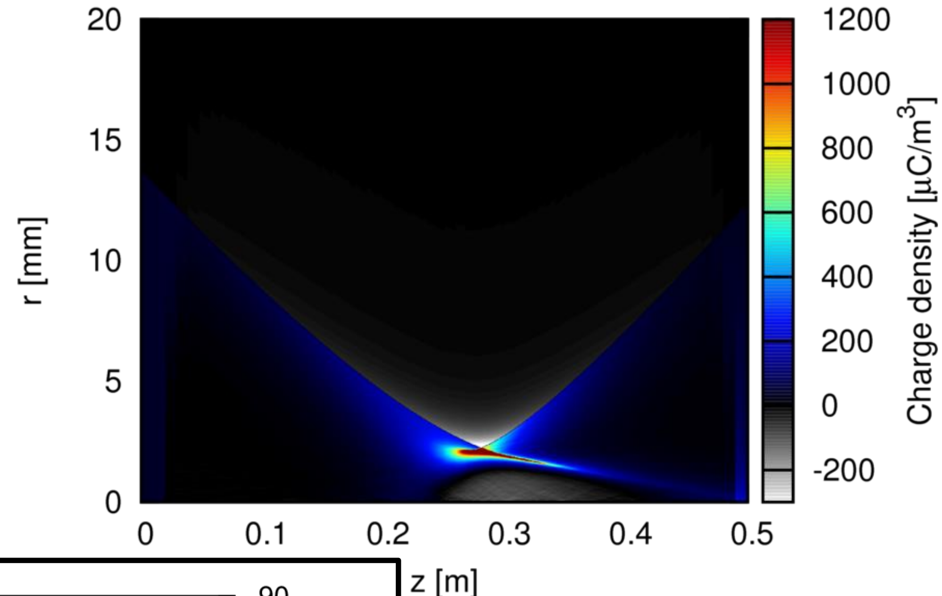


50 cm beam transport, 120 keV, 50 mA

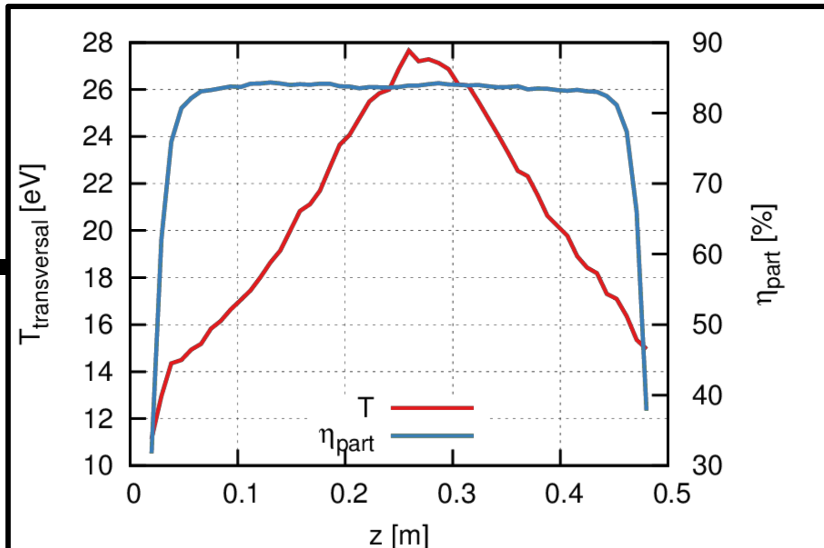
Results for the Drift System

Comparison to bender simulation





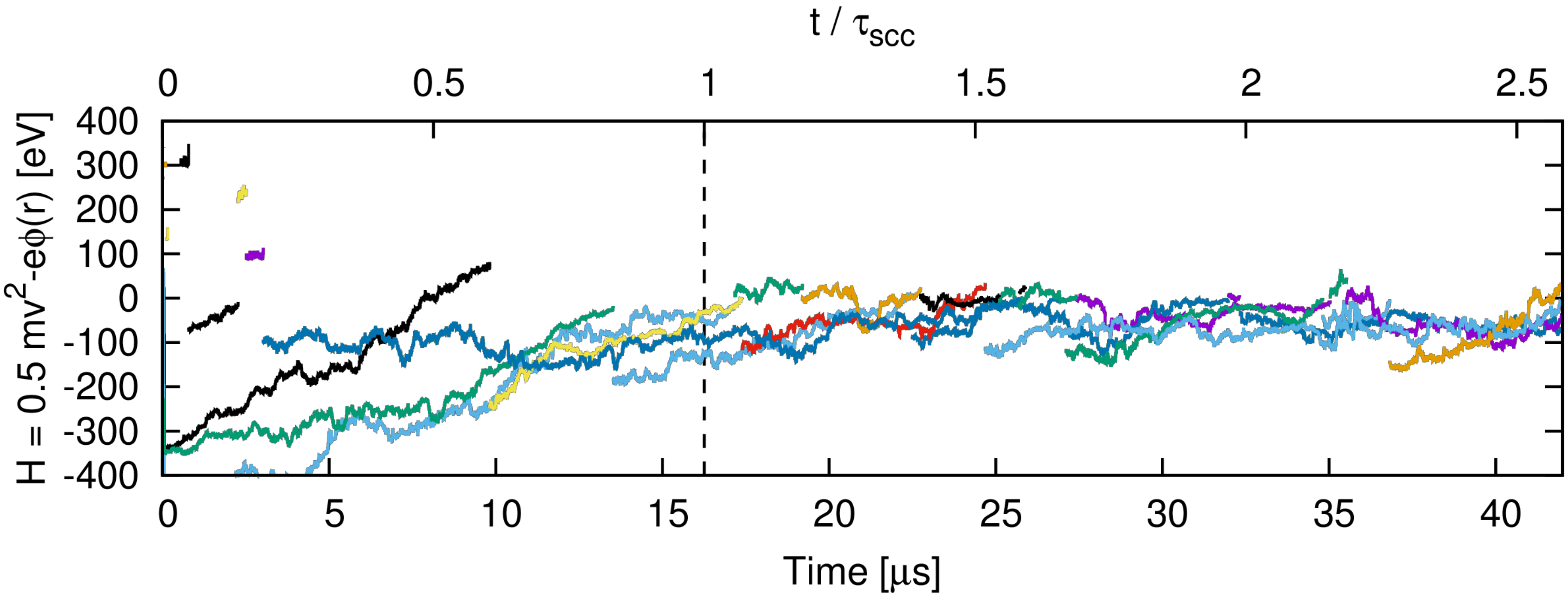
7 days, 24 cores



2 hours, 1 core

Origin of the „Thermalization“

- Energy of random electron tracks over time:

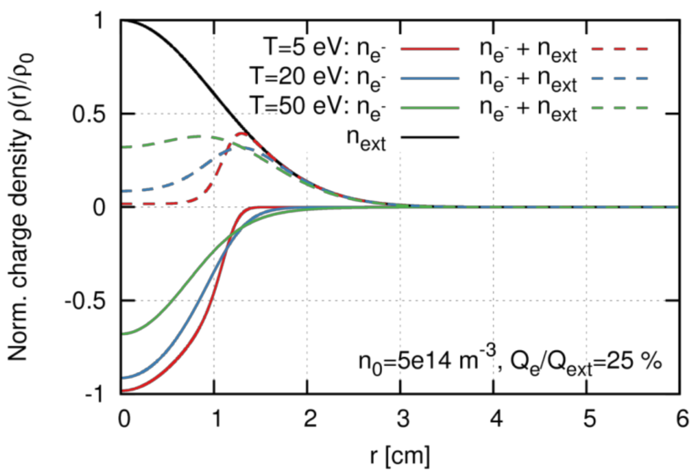


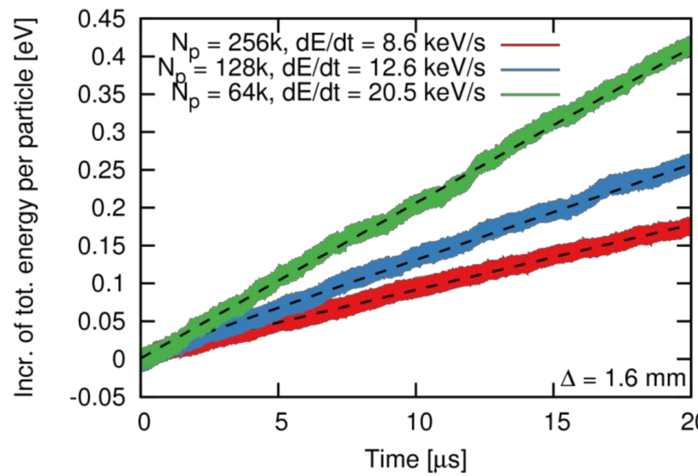
- Random walk until H > 0, then get gradually lost
- Is energy conserved in the simulation?

Origin of the „Thermalization“

Stochastic heating in a test system

$f(\mathbf{r},\mathbf{p}) = f_0 \exp\left(-\frac{H}{k_b T}\right)$ is a solution of $\underbrace{\frac{\partial f}{\partial t}}_{=0} + \frac{\mathbf{p}}{m}\frac{\partial f}{\partial q} + qE\frac{\partial f}{\partial p} = 0\,.$

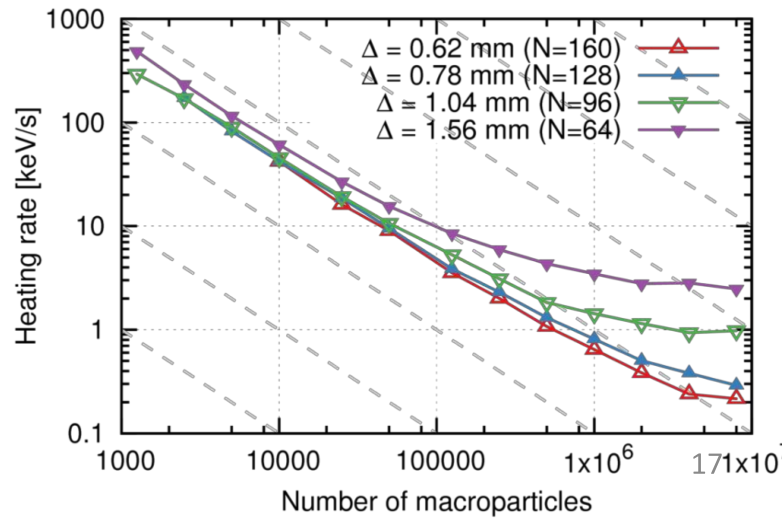




- Stochastic heating [1]:
  - „Error“ field with $\overline{\delta E} = 0, \overline{\delta E^2} \neq 0$

    $\Delta T = T_{i+1} - T_i = \frac{1}{2}\frac{q^2}{m}\overline{\delta E^2}\Delta t^2$
  - Effect from particle statistics:

    $|\delta E| \backsim N^{-1/2}$



[1] Hockney, Eastwood – Computer Simulation Using Particles, 1989

Origin of the „Thermalization“

- Dependence on the number of simulation particles
- Temperatures linked to compensation degree
- Not responsible:
  - Secondary electron energy distribution
  - Coulomb collisions

Hypothesis:

Stochastical heating

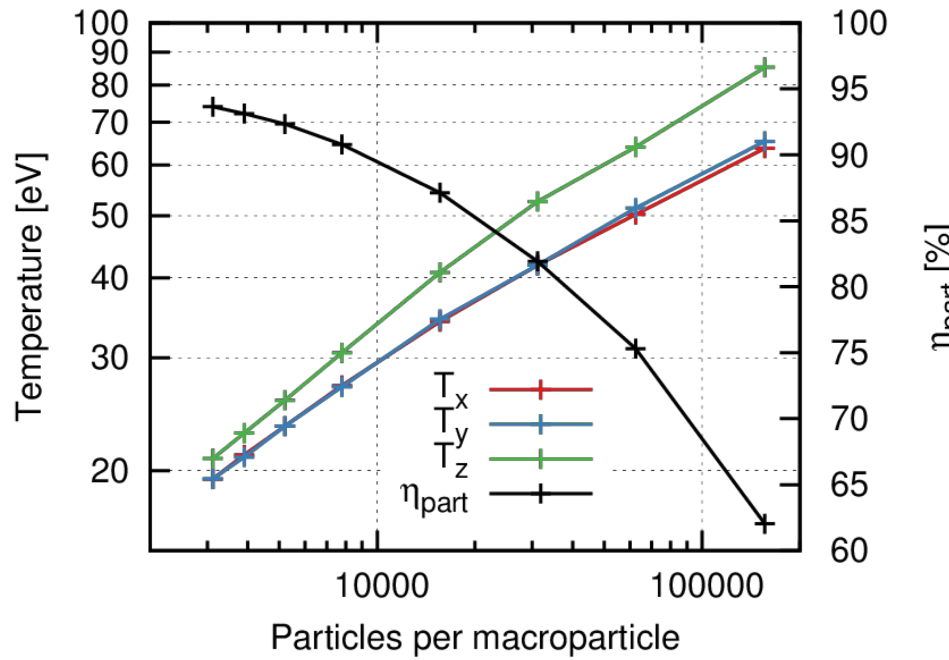
↓

Particle losses compensate increased energy contribution

↓

Gaussian distributions & $\eta < 100$ %

Thermalization



- Further indications:
  - 1d simulations show almost no „temperature“
  - Simulation with static beam show lower temperatures

Conclusions & Outlook

- Space-charge compensation was included in a self-consistent way
  - Electrons follow a Boltzmann distribution
  - The dynamics are completely determined by the plasma nature of the compensation electrons
  - Hypothesis was formed: thermalization is a result of stochastical heating
- Before physical heating processes can be included

$$P_{\mathrm{Heating}} \cong \frac{e^2}{4\pi\varepsilon_0^2 m_e} \frac{n_{\mathrm{beam}} q_{\mathrm{beam}}^2}{v_{\mathrm{beam}}} \ln(\Lambda) \quad [1] \xrightarrow{T=20\ \mathrm{eV}} \approx 60\ \mathrm{keV/s}$$

numerical effects need to be removed… how?

[1] R. Dölling, Raumladungskompensation driftender intensiver Strahlen niederenergetischer Ionen und Techniken zu ihrer Vermessung, Dissertation, Goethe Universität Frankfurt am Main 2000

Conclusions & Outlook

- Space-charge compensation was included in a self-consistent way
  - Electrons follow a Boltzmann distribution
  - The dynamics are completely determined by the plasma nature of the compensation electrons
  - Hypothesis was formed: thermalization is a result of stochastical heating
- Before physical heating processes can be included

$$P_{\mathrm{Heating}} \cong \frac{e^2}{4\pi\varepsilon_0^2 m_e} \frac{n_{\mathrm{beam}} q_{\mathrm{beam}}^2}{v_{\mathrm{beam}}} \ln(\Lambda) \quad [1] \xrightarrow{T=20\ \mathrm{eV}} \approx 60\ \mathrm{keV/s}$$

numerical effects need to be removed… how?

Thank you for your attention!